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A STUDY OF THE PROPERTIES OF
A NEW GOODNESS-OF-FIT TEST

by

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and

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Monterey, California

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We investigate the power properties of a new goodness-of-fit test proposed by Foutz (1980). This new test is compared with the Chi squared test and the Kolmogorov-Smirnov (K-S) test for normality when the samples come from (i) the family of asymmetric stable distributions, (ii) mixtures of normal distributions, and (iii) the Pearson family. The general conclusion is that the new test performs better than the Chi squared and the K-S test when the parent distribution is heavy-tailed. If the hypothesized distribution differs from the true distribution in location only, the new test does not do as well as the other two.

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1. INTRODUCTION

In a recent article Foutz (1980) introduced a new test for goodness-of-fit, to be called the F_n test in the sequel. Although the test was proposed for fitting a continuous p-variate distribution, it applies equally well to univariate problems. The null distribution of the test statistic was shown to be distribution free as well as being independent of the number of variates p. Foutz obtained an integral representation for the null CDF of F_n ; explicit expressions for this CDF were given for sample sizes 2 and 3. Closed form solutions for the CDF for larger sample sizes are quite hard to derive and Foutz has provided a large sample normal approximation to the null distribution of F_n .

In a preliminary comparison with ten replications of 50 simulated samples from (i) a mixture of uniform distributions and (ii) a standard normal distribution, Foutz found that the F_n test outperformed both the Chi squared test and the Kolmogorov-Smirnov (K-S) test.

In this paper we present the results of an extensive investigation to compare the three goodness-of-fit tests, the Chi squared test, the K-S test and the F_n test. Members from three families of distributions, viz., the family of asymmetric stable distributions, mixtures of normals, and the Pearson family have been selected to represent the true underlying distribution of the samples. The goodness-of-fit tests are applied to test the hypothesis that the samples are from a standard normal distribution. The measure of comparison used in the study is the empirical power, based on 5000 replications of each of the tests.

The Chi squared and Kolmogorov-Smirnov statistics were computed using the IMSL⁺ routines GFIT and NKS1. The number of cells used in the Chi

⁺International Mathematical and Statistical Libraries, 7500 Bellaire Boulevard, Houston, TX 77036

squared computation was 6, 8, and 8 for sample sizes of 20, 30, and 50, respectively.

A brief discussion of the F_n test is given in Section 2 and a description of the simulation is in Section 3. The results of the simulation are presented in Section 4. FORTRAN codes used for the simulation and detailed tables of simulation results are in Appendices I and II.

2. F_n TEST

The F_n test is based on a comparison of a continuous empirical distribution function (CEDF) with the hypothesized CDF. The CEDF is obtained by "spreading" the total mass over "statistically equivalent blocks" generated by the sample. As shown in Anderson (1966) and Foutz (1980), given the order statistics of a random sample of size $(n-1)$, n statistically equivalent blocks that partition the sample space can be constructed in many different ways by choosing what are called cutting functions. An intuitively appealing set of blocks, which is the one used in this study is obtained by choosing the identity functions for the cutting functions. In this case, the n statistically equivalent blocks are $B_1 = (-\infty, x_{(1)}]$, $B_2 = (x_{(1)}, x_{(2)}]$, ..., $B_n = (x_{(n-1)}, \infty)$ where $x_{(1)}, x_{(2)}, \dots, x_{(n-1)}$ are the order statistics of a sample of size $n-1$. The CEDF is constructed by spreading a mass $\frac{1}{n}$ continuously and in the same proportion as the hypothesized CDF over each block. If H_0 is the hypothesized CDF and \hat{H}_n the CEDF, the test statistic F_n is defined as

$$F_n = \sup_x |H_n(x) - H_0(x)| \quad (1)$$

Let D_i , $i = 1, 2, \dots, n$ be the probability contents of the blocks B_i under the null CDF H_0 , i.e., $D_i = P[x \in B_i | H_0]$. A computationally convenient form for F_n can be shown to be

$$F_n = \sum_{i=1}^n \max(0, \frac{1}{n} - D_i) \quad (2)$$

Foutz has provided the null distribution of F_n in integral form and derived closed form solutions for $n = 3, 4$. Simplifying the expressions given by Foutz for $n = 3, 4$ yields

$$P(F_3 \leq x) = \begin{cases} 6x^2 & 0 \leq x \leq \frac{1}{3} \\ 1 - 3(\frac{1}{2} - x)^2 & \frac{1}{3} < x \leq 2/3 \\ 1 & x > 2/3 \end{cases} \quad (3)$$

and

$$P(F_4 \leq x) = \begin{cases} 20x^3 & 0 \leq x \leq \frac{1}{4} \\ -20x^3 + 18x^2 - \frac{9}{4}x + \frac{1}{16} & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 1 - 4(\frac{3}{4} - x)^3 & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 1 & x > \frac{3}{4} \end{cases} \quad (4)$$

We have also obtained the expression for the null CDF for $n = 5$,

$$P(F_5 \leq x) = \begin{cases} 70x^4 & 0 \leq x \leq \frac{1}{5} \\ -105x^4 + 80x^3 - 12x^2 + \frac{16}{25}x - \frac{1}{125} & \frac{1}{5} < x \leq \frac{2}{5} \\ 45x^4 - 80x^3 + \frac{228}{5}x^2 - \frac{176}{25}x + \frac{31}{125} & \frac{2}{5} < x \leq \frac{3}{5} \\ 1 - 5(\frac{4}{5} - x)^4 & \frac{3}{5} < x \leq \frac{4}{5} \\ 1 & x > \frac{4}{5} \end{cases} \quad (5)$$

As is evident the exact distribution is quite difficult to obtain for higher sample sizes. A large sample normal approximation, due to Foutz is given by

$$\lim_{n \rightarrow \infty} P\left[F_n \leq x\right] = \Phi\left[\frac{n(x - e^{-1})}{(2e^{-1} - 5e^{-2})^{1/2}}\right] \quad (6)$$

where Φ is the standard normal CDF. For $n-1 = 20, 30, 50$ we used this approximation to test the hypothesis that a simulated sample from $U[0,1]$, a uniform distribution on $[0,1]$, is in fact from that distribution. In 80,000 replications, the observed significance level (number of hypothesis rejections/80,000) was consistently smaller than the nominal value as can be seen from

TABLE 1

EMPIRICAL SIGNIFICANCE LEVEL OF
FOUTZ F_n TEST USING ASYMPTOTIC APPROXIMATION
(80,000 REPLICATIONS)

Sample Size	20	30	50
Significance Level			
.10	.0757	.0800	.0859
.05	.0372	.0399	.0428
.01	.0082	.0083	.0093

Table 1. We therefore constructed a Monte Carlo CDF of F_n for $n-1 = 2, 3, 4, 20, 30, 50$ based on 25,000 computer generated F_n values; these represent values of the F_n statistic for testing the hypothesis that a set of samples from $U[0,1]$ is in fact from $U[0,1]$. A comparison of the Monte Carlo CDF with the exact CDF for $n-1 = 2, 3, 4$ is provided in Table 2. It can be seen that the Monte Carlo CDF provides a reasonable approximation even for small n .

The power properties of the F_n test detailed in this paper are based on the Monte Carlo CDF of F_n . Critical values obtained from the Monte Carlo simulation for significance levels .01, .05, .1 and $n-1 = 20, 30, 50$ are in Table 3. In Table 4 we present the observed significance level in 225,000 replications when testing if a set of samples from $U[0,1]$ is in fact from $U[0,1]$.

TABLE 2

MONTE CARLO SIMULATION VS EXACT VALUES OF CDF

x	n=3		n=4		n=5	
	MC	EXACT	MC	EXACT	MC	EXACT
.40	.7883	.7867	.7624	.7625	.7607	.7600
.45	.8604	.8592	.8725	.8725	.8671	.8693
.50	.9182	.9167	.9396	.9375	.9405	.9405
.55	.9598	.9592	.9694	.9680	.9768	.9778
.60	.9864	.9867	.9860	.9865	.9915	.9920
.65	.9989	.9991	.9952	.9960	.9975	.9975

TABLE 3

CRITICAL VALUES FOR F_n TEST
OBTAINED BY MONTE CARLO SIMULATION

Sample Size	20	30	50
Significance Level			
.10	.42714	.41903	.40816
.05	.44865	.43553	.42116
.01	.48659	.46579	.44487

TABLE 4

EMPIRICAL SIGNIFICANCE LEVEL OF
FOUTZ F_n TEST USING MONTE CARLO APPROXIMATION
(225,000 REPLICATIONS)

Sample Size	20	30	50
Significance Level			
.10	.1006	.0970	.1003
.05	.0486	.0486	.0498
.01	.0103	.0101	.0102

3. DESCRIPTION OF SIMULATION

In our simulation we generated deviates from three families of distributions. The family of asymmetric stable distributions has been previously used by Saniga and Miles (1979) to investigate the power of several goodness-of-fit tests. We used the same set of parameter values, $\alpha = 1.0(.3)1.9$ and $\beta = 0(.25)1.00$ they used. Mixed normal distributions have often been used for such tests, and we have included a family which is a composite of $N(0,1)$ and $N(0,\sigma)$, $\sigma \neq 1$, and another set which is a composite of $N(.5,1)$ and $N(0,\sigma)$. Pearson distributions considered include a variety of shapes, from "near" normal to U and J shaped. Discussion of the procedures used to generate pseudorandom deviates from each of these families follows.

The sample data was obtained by starting with one or more uniformly distributed pseudorandom deviates. These were generated using the IMSL subroutine GGUBS, the basis of which is discussed in Lewis, Goodman, and Miller (1969).

3.1 Generation of Asymmetric Stable Deviates (Random Stabilized Standard Form)

This family of distributions contains as a special case the normal distribution ($\alpha = 2$, $\beta = 0$) and the Cauchy distribution ($\alpha = 1$, $\beta = 0$). For $\alpha < 2$ the distributions have infinite variance, which makes them useful for determining the ability of a goodness-of-fit test to detect heavy tailed distributions. The deviates were generated using the program given in Chambers, Mallows, and Stuck (1976). This subroutine, RSTAB, used one deviate from $U[0,1]$ and one exponentially distributed deviate to generate one RSSF deviate.

3.2 Generation of Mixed Normal Deviates

Mixed normals of the form $(1-\gamma)N(\mu_1, \sigma_1) + \gamma N(\mu_2, \sigma_2)$ were generated using the IMSL routine MDNRIS to convert uniform samples into standard normal samples. To obtain a set of N mixed normal variates we proceeded as follows: (i) $2N$ uniform random variates $\{u_i\}$ were generated using GGUBS; (ii) for each $i = 1, \dots, N$, MDNRIS was used to convert u_i to a standard normal z_i . z_i was then transformed to a normal with mean μ_1 and variance σ_1 , or with mean μ_2 and variance σ_2 , depending on whether $u_{i+N} \geq \gamma$, or $u_{i+N} < \gamma$, respectively.

3.3 Generation of Pearson Type I, II Deviates

The generation of samples from Pearson Type I and II distributions was done via table look up and linear interpolation on the inverse cumulative distribution function. Sufficient entries to assure four significant decimal places in the final answer was achieved adaptively using numerical integration. Before discussion of the precise details of the process, we digress for a discussion of the Pearson distributions, and particularly types I and II.

Following Johnson and Kotz (1970), the Pearson probability density function $p(x)$ is given by

$$\frac{1}{p} \frac{dp}{dx} = \frac{a + x}{c_0 + c_1 x + c_2 x^2}.$$

It can be shown that a , c_0 , c_1 , and c_2 can be expressed in terms of non-negative parameters β_1 , β_2 , and the variance μ_2 , obtaining

$$\begin{aligned} c_0 &= (4\beta_2 - 3\beta_1)(10\beta_2 - 12\beta_1 - 18)^{-1} \mu_2 \\ a = c_1 &= \sqrt{\beta_1} (\beta_2 + 3)(10\beta_2 - 12\beta_1 - 18)^{-1} \sqrt{\mu_2} \\ c_2 &= (2\beta_2 - 3\beta_1 - 6)(10\beta_2 - 12\beta_1 - 18)^{-1}. \end{aligned} \tag{7}$$

Type I distributions are characterized by

$$\kappa = \frac{1}{4} c_1^2 (c_0 c_2)^{-1} < 0 ,$$

while Type II have $\kappa = 0$ with $\beta_1 = 0$, $\beta_2 < 3$. For Type I and II the probability density function is of the form

$$p(x) = K(x - a_1)^{m_1} (a_2 - x)^{m_2} . \quad (8)$$

Generation of a sequence of pseudorandom Pearson Type I and II deviates involves the following steps: (i) generate a sequence of deviates $\{u_i\}$, from $U[0,1]$; (ii) transform these to Pearson deviates by finding v_i so that

$$\int_{a_1}^{v_i} p(x) dx = u_i .$$

This necessitates being able to efficiently obtain the

inverse CDF, i.e., if $F(v) = \int_{a_1}^v p(x) dx$, then we need $v_i = F^{-1}(u_i)$. We

will denote F^{-1} by G in order to simplify notation.

Representation of the inverse CDF, $G(s)$, was achieved by linear interpolation in a table generated by numerical integration of $p(x)$. The adaptive process used to assure four decimal place accuracy, i.e., magnitude of the error less than $.5 \times 10^{-4}$ is described now.

The error in linear interpolation between points (s_i, G_i) and (s_{i+1}, G_{i+1}) is no more than $\frac{1}{8}(s_{i+1} - s_i)^2 G_2$, where $G_2 = \max_{s_i \leq s \leq s_{i+1}} |G''(s)|$.

Since the intervals will be small, we can approximate

$$G_2 \text{ by } \frac{G'(s_{i+1}) - G'(s_i)}{s_{i+1} - s_i} , \quad \text{and then using the fact that}$$

$$G'(s) = (F^{-1}(s))' = \frac{1}{F'(G)} = \frac{1}{p(G)} , \text{ we obtain}$$

$$G_2 \approx \frac{\frac{1}{p(G_{i+1})} - \frac{1}{p(G_i)}}{s_{i+1} - s_i}.$$

Thus, an error estimate in (s_i, s_{i+1}) is given by

$$\left| \frac{\frac{1}{p(G_{i+1})} - \frac{1}{p(G_i)}}{s_{i+1} - s_i} \right| \frac{(s_{i+1} - s_i)^2}{8} = \frac{1}{8} \left| \frac{1}{p(G_{i+1})} - \frac{1}{p(G_i)} \right| (s_{i+1} - s_i). \quad (9)$$

Potential problems occur if $p(G) = 0$, as may happen at a_1 and a_2 . Since we require error less than $.5 \times 10^{-4}$, it is clear this must be the case when $|G_{i+1} - G_i| < .5 \times 10^{-4}$; hence if $p(G)$ is very small in the interval (s_i, s_{i+1}) we have an alternative scheme for accepting an interval, one which came into play for ranges where p is small.

These ideas were the basis for adaptive construction of a suitable table (s_i, G_i) for the inverse cumulative distribution function. We have

$s_0 = 0, G_0 = a_1$. We describe the general scheme for obtaining (s_{i+1}, G_{i+1}) given (s_i, G_i) and note how an estimate of G_{i+1} is generated afterwards. Given an estimate for G_{i+1} , the value of $\Delta s_i = s_{i+1} - s_i$ is obtained by numerical

numerical integration of $\int_{G_i}^{G_{i+1}} p(x) dx$. This is accomplished by the

adaptive quadrature routine, DCADRE, from the IMSL library. If m_1 or m_2 are negative, subtracting out the singularity was used for intervals near a_1 and a_2 , respectively. The double precision version of DCADRE is used and an absolute error tolerance of 10^{-6} is requested. The routine returns Δs_i and the error estimate (9) is computed. If it is less than $.5 \times 10^{-4}$, the result is accepted, we set $s_{i+1} = s_i + \Delta s_i$, and proceed to the next interval. Suppose the error estimate, $E_{est} > .5 \times 10^{-4}$. Since $E_{est} = O((\Delta G)^2)$, and

$O(\Delta G_i) = O(\Delta s_i)$, we obtain a new estimate for ΔG_i by taking it to be

$$\frac{\Delta G_i}{2} \left(\sqrt{\frac{.25 \times 10^{-4}}{E_{\text{est}}}} + 1 \right).$$

This takes the error based on the new ΔG_i to approximately midway between its current value and $.25 \times 10^{-4}$. More than one correction of this sort may be required, in particular initially where a reasonable estimate of ΔG_0 is not available.

Once an interval has been accepted (or rather, a point (s_i, G_i)) we increment the interval counter i , and then estimate the new interval size ΔG_i by $\frac{\Delta G_{i-1}}{2} \left(\sqrt{\frac{.475 \times 10^{-4}}{E_{\text{est}}}} + 1 \right)$. This yields, based on the above assumptions, a ΔG_i which should give an error for the next interval which is midway between the previous one and $.475 \times 10^{-4}$.

An initial value for ΔG_0 is required. Although not very sophisticated, we simply take $\Delta G_0 = \frac{a_2 - a_1}{1000}$, and depend on the adaptive machinery described above to decrease it to meet the error tolerance, or increase successive intervals as required for efficient representation.

It is true that the final value of s_i , call it s_N , should be equal to one. Because of the numerical integration, this is never exactly the case. In the worst case, $\beta_1 = .01$, $\beta_2 = 1.9$, we obtained the final value $s_N \approx .9999999845$ an error of about 1.55×10^{-8} . In order to avoid any problems due to the table not covering $[0,1]$ exactly the simplest procedure was to replace each computed s_i by s_i/s_N , thus distributing the error over the entire interval and yielding a consistent table. Note that this is well within the error tolerance of $.5 \times 10^{-4}$.

The procedure has been thoroughly tested for its efficiency in representing the inverse CDF as well as for accuracy. For the particular cases

in which we were interested, the inverse CDF was represented by a table with no more than 729 entries. This occurred for the case $\beta_1 = .25$, $\beta_2 = 3.2$, which has an inverse CDF with very large slopes. For more gently sloping inverse CDF's we were able to use as few as 101 intervals, as in the case $\beta_1 = .01$, $\beta_2 = 1.75$. Most intervals had an error estimate of between $.4 \times 10^{-4}$ and $.5 \times 10^{-4}$, which shows that the interval sizing process we have used worked quite efficiently, with few initial estimates being rejected for being too large, without also resulting in intervals much too small.

The routine was checked against the published tables of Johnson, Nixon, Amos and Pearson (1963) for many values of the parameters and at most of the percentage points. With a few exceptions, where a difference of one in the fourth decimal place was noted, the results check exactly. Generally in these cases the fifth place was four or five so that the actual error was probably well within our tolerance.

4. RESULTS

The results of the simulation are summarized in Tables 5-12. The empirical power, in 5000 replications, of the Foutz F_n test compared with that for the Chi squared test or the K-S test is presented as a percent improvement in the probability of rejecting the null hypothesis that the distribution of the samples is the standard normal. A negative entry means that the power of the F_n test was smaller than that for the Chi squared test or the K-S test, whichever is appropriate.

The simulation has revealed that the F_n test is better than the Chi squared test which in turn is better than the K-S test when the true distribution of the samples is heavy tailed. Many such distributions are included in the asymmetric stable family as well as the family of mixtures of normals. For the mixed normal family, if the two normals involved in the mixture differ in the means the K-S test performed better than the F_n test even when the variances differed. We now discuss in more detail the results for each of the three families of distributions.

4.1 Asymmetric Stable Family

The results for the F_n test versus the Chi squared test are summarized in Table 5. The F_n test outperformed the Chi squared test for n equal to 21 and 31. When $n = 51$, as $\alpha + \beta$ increased the performance of the F_n test deteriorated as can be seen from the lower right part of Table 5. Another general observation is that as the significance level is decreased the improvement in power for the F_n test is accentuated.

The comparative figures for the F_n test versus the K-S test are presented in Table 6. Here again the F_n test did much better than the K-S test;

the results also indicate that for the asymmetric stable family the Chi squared test has a higher power than the K-S test.

4.2 Mixtures of Normals

The mixed normal distributions that we considered were of two basic types. The first type is of the form $(1-\gamma)N(0,1) + \gamma N(0,\sigma)$ with $\sigma = 2,3,4$ and $\gamma = .1, .2, .3, 1.0$; note that when $\gamma = 1.0$ the distribution is not a true mixture but $N(0,\sigma)$. The second type is a mixture of the form $(1-\gamma)N(.5,1) + \gamma N(0,\sigma)$ with $\sigma = 3, \gamma = .2, .3$ and $\sigma = 4, \gamma = .2$.

The F_n test did significantly better than the Chi squared test except for $n = 51$ and $\gamma = 1.0$ (see Table 7); in the latter case the Chi squared test turned out to be the better of the two tests.

When the F_n test was compared with the K-S test the F_n test turned out to be consistently better (Table 8).

In the case of a mixture of the second type, which included a location shift (Tables 9, 10), the K-S test proved to be superior to the F_n test while the comparison between the F_n test and the Chi squared test appeared to be inconclusive.

4.3 Pearson Family

We chose ten distributions of types I and II to encompass a variety of shapes as shown in figure 1; the standard normal is superimposed as a dotted curve in each of the graphs. The comparison of the F_n test versus the Chi squared test proved to be inconclusive. However, the K-S test appeared to have a higher power than the F_n test when the shape of the distribution was "near" normal such as the ones for $(\beta_1, \beta_2) = (0, 2.3), (0, 2.8)$ and $(.25, 3.2)$.

FIGURE 1

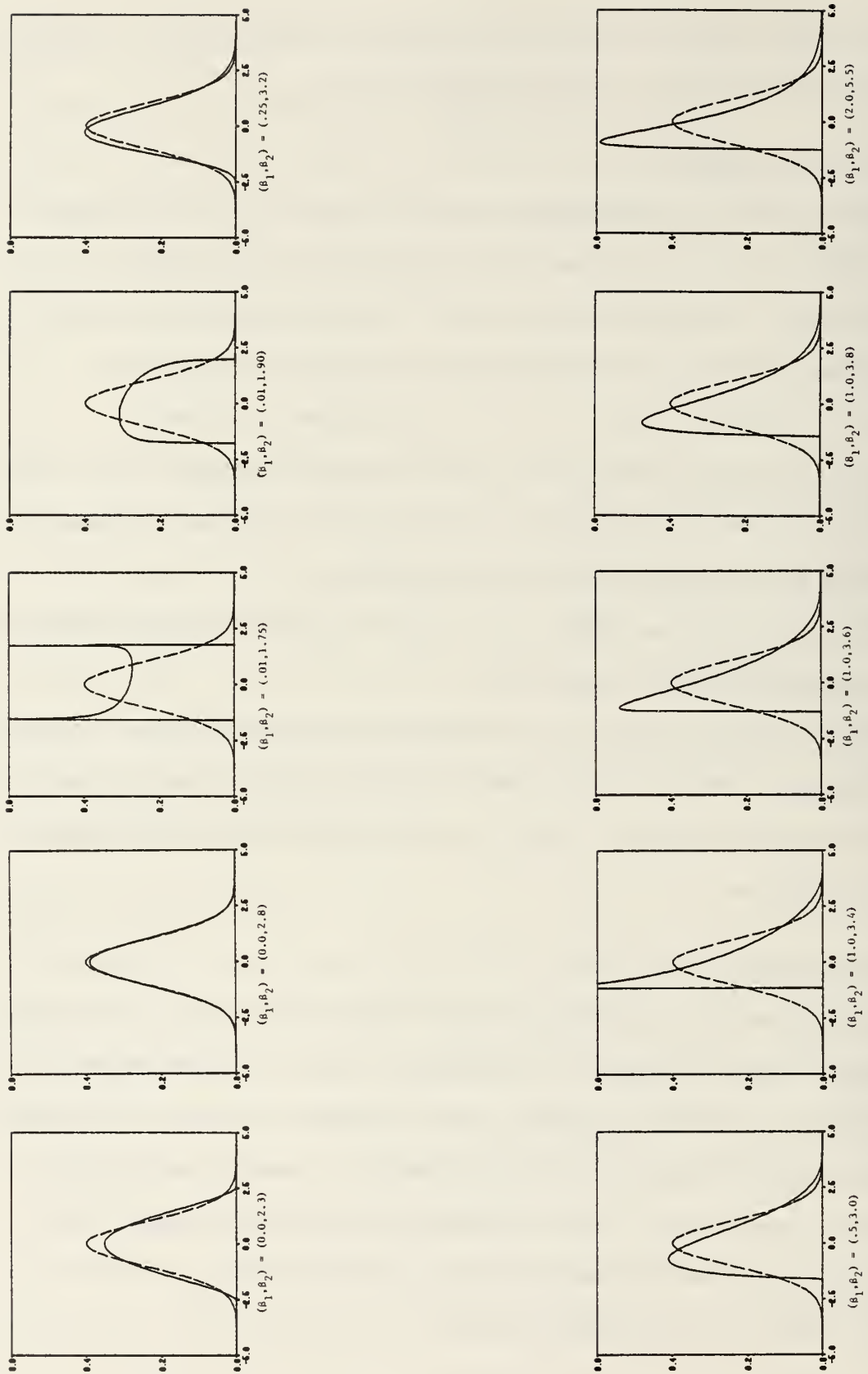


TABLE 5

FOUTZ F_n TEST VS CHI SQUARED TEST
RANDOM STABILIZED STANDARD DISTRIBUTIONS

		$\beta \rightarrow$	0.0	0.25	0.50	0.75	1.00
Sig Level	α						
n=21	.10	1.0	102.0	102.7	101.6	95.8	96.9
		1.3	96.1	88.6	69.1	49.1	11.7
		1.6	72.0	68.3	59.5	49.0	29.2
		1.9	54.3	55.4	55.4	53.9	55.8
	.05	1.0	120.2	117.2	121.6	114.9	110.3
		1.3	97.9	98.2	71.2	49.6	- 1.4
		1.6	71.2	63.4	58.6	41.4	20.1
		1.9	49.8	45.6	51.1	49.8	44.2
	.01	1.0	256.0	215.9	238.0	234.5	233.8
		1.3	174.8	158.6	131.8	71.7	- 8.0
		1.6	118.5	130.3	107.9	62.1	21.7
		1.9	81.6	73.0	99.0	89.7	78.7
n=31	.10	1.0	37.9	36.8	38.1	39.3	37.0
		1.3	35.9	34.0	28.4	13.8	-10.4
		1.6	26.6	26.2	16.5	11.1	0.9
		1.9	17.6	19.5	19.4	13.4	16.7
	.05	1.0	47.4	44.6	46.6	50.2	46.7
		1.3	41.8	38.9	31.5	11.7	-18.6
		1.6	28.8	25.7	13.0	5.2	- 7.6
		1.9	12.0	15.8	13.3	8.4	14.4
	.01	1.0	74.7	75.2	73.8	73.5	70.2
		1.3	55.8	46.2	31.2	3.9	-33.7
		1.6	32.2	30.7	11.0	- 6.0	-21.3
		1.9	14.5	18.1	10.1	8.0	1.0
n=51	.10	1.0	11.4	13.3	11.1	10.6	11.4
		1.3	9.6	12.3	6.1	- 1.1	-11.1
		1.6	1.2	- 0.5	- 4.9	- 7.3	-13.9
		1.9	-10.8	- 8.9	-11.0	-11.1	-11.2
	.05	1.0	17.4	18.5	17.1	16.3	17.1
		1.3	12.6	12.2	5.0	-3.5	-18.2
		1.6	0.5	- 3.5	- 8.7	-12.0	-21.7
		1.9	-14.3	-12.9	-15.1	-16.3	-15.9
	.01	1.0	30.5	30.5	31.2	28.4	25.8
		1.3	14.1	13.8	3.1	-12.2	-35.2
		1.6	- 5.4	- 8.3	-20.6	-23.5	-37.0
		1.9	-24.2	-24.2	-24.1	-24.2	-26.1

TABLE 6

FOUTZ F TEST VS KOLMOGOROV-SMIRNOV TEST RANDOM
RANDOM STABILIZED STANDARD DISTRIBUTIONS

		$\beta \rightarrow$	0.0	0.25	0.50	0.75	1.00
	Sig Level	α					
n=21	.10	1.0	120.2	123.8	110.0	120.4	115.2
		1.3	102.8	100.6	76.0	43.5	0.9
		1.6	87.7	77.2	61.3	49.6	22.7
		1.9	65.5	61.7	62.9	65.1	57.8
	.05	1.0	197.5	196.0	186.7	185.9	192.1
		1.3	170.8	151.2	111.3	68.1	- 2.8
		1.6	131.3	110.4	89.9	67.0	30.4
		1.9	83.7	90.5	87.5	85.6	74.4
	.01	1.0	601.5	465.5	530.4	522.7	526.8
		1.3	396.4	338.1	245.0	121.4	- 0.6
		1.6	305.7	243.9	165.2	117.7	45.6
		1.9	140.1	147.5	176.2	176.5	163.8
n=31	.10	1.0	82.1	80.6	83.3	84.7	81.6
		1.3	83.6	77.6	56.7	25.5	- 9.2
		1.6	63.8	59.5	42.6	29.4	7.3
		1.9	49.5	55.6	48.2	44.8	40.3
	.05	1.0	150.9	149.8	145.6	152.7	151.5
		1.3	147.3	129.2	88.1	40.5	- 13.0
		1.6	107.8	98.9	65.0	41.1	11.4
		1.9	77.2	86.7	69.5	72.5	68.2
	.01	1.0	428.5	429.5	447.0	399.0	471.3
		1.3	386.1	316.4	190.6	76.4	- 16.2
		1.6	267.3	210.3	136.9	79.9	24.1
		1.9	190.8	186.2	153.0	146.8	136.8
n=51	.10	1.0	38.5	42.7	38.9	37.5	37.7
		1.3	49.3	49.4	29.6	8.4	- 10.3
		1.6	40.0	35.7	24.0	11.4	- 6.7
		1.9	24.4	31.4	26.1	22.2	19.5
	.05	1.0	77.6	81.2	78.0	78.3	76.0
		1.3	94.4	85.2	53.2	14.7	- 16.4
		1.6	79.4	61.4	37.9	17.0	- 8.1
		1.9	49.8	54.6	50.6	42.0	35.4
	.01	1.0	285.4	287.0	275.2	270.9	262.6
		1.3	287.6	243.0	138.7	37.7	- 26.1
		1.6	196.2	173.3	91.2	41.0	- 7.9
		1.9	130.4	126.3	115.4	111.8	99.6

TABLE 7

FOUTZ F_n TEST VS CHI SQUARED TEST
MIXED NORMAL DISTRIBUTIONS

		$\gamma \rightarrow$	1.00	0.30	0.20	0.10
	Sig Level	N(0, σ)				
n=21	.10	N(0,2)	55.8	69.3	76.3	40.7
		N(0,3)	32.8	86.8	75.9	44.7
		N(0,4)	20.5	101.4	89.0	75.7
	.05	N(0,2)	47.0	46.4	64.5	23.7
		N(0,3)	30.5	71.3	69.0	34.7
		N(0,4)	22.6	91.1	76.8	48.1
	.01	N(0,2)	62.0	262.1	107.5	88.6
		N(0,3)	61.2	109.8	145.8	86.3
		N(0,4)	49.0	118.3	131.1	125.0
n=31	.10	N(0,2)	13.8	46.9	42.4	22.0
		N(0,3)	4.3	56.7	50.6	51.5
		N(0,4)	1.8	55.8	58.7	39.2
	.05	N(0,2)	12.0	38.3	29.2	28.6
		N(0,3)	3.1	53.2	48.4	53.2
		N(0,4)	1.0	57.4	61.5	44.4
	.01	N(0,2)	1.2	62.1	46.0	52.1
		N(0,3)	0.6	47.3	103.1	104.7
		N(0,4)	0.1	84.0	95.5	61.5
n=51	.10	N(0,2)	-13.1	23.6	26.1	9.7
		N(0,3)	- 5.9	19.3	28.4	25.7
		N(0,4)	- 1.4	26.2	29.5	35.6
	.05	N(0,2)	-16.2	28.9	45.5	17.4
		N(0,3)	- 9.4	29.2	43.4	38.0
		N(0,4)	- 2.5	34.1	45.4	39.7
	.01	N(0,2)	-26.9	83.8	54.8	67.4
		N(0,3)	-17.7	41.6	45.9	67.9
		N(0,4)	- 7.9	24.5	78.3	51.4

TABLE 8

FOUTZ F_n TEST VS KOLMOGOROV-SMIRNOV TEST
MIXED NORMAL DISTRIBUTIONS

		$\gamma \rightarrow$	1.00	0.30	0.20	0.10
Sig Level		N(0, σ)				
n=21	.10	N(0,2)	60.5	36.1	31.8	8.7
		N(0,3)	52.6	59.3	42.9	19.3
		N(0,4)	38.8	71.5	54.0	40.2
	.05	N(0,2)	87.4	49.5	36.4	6.8
		N(0,3)	91.8	75.4	49.8	16.9
		N(0,4)	74.4	90.2	66.6	43.7
	.01	N(0,2)	156.3	94.4	43.1	15.8
		N(0,3)	223.6	112.3	71.0	41.8
		N(0,4)	198.1	107.0	101.2	26.6
n=31	.10	N(0,2)	42.6	19.3	27.8	13.9
		N(0,3)	31.3	52.9	38.8	24.1
		N(0,4)	18.5	66.2	52.9	21.1
	.05	N(0,2)	69.3	29.9	27.8	20.6
		N(0,3)	60.8	65.7	52.9	36.8
		N(0,4)	38.7	101.4	61.9	36.2
	.01	N(0,2)	135.8	52.9	67.3	55.3
		N(0,3)	183.9	107.5	120.3	66.0
		N(0,4)	129.8	163.9	152.2	72.1
n=51	.10	N(0,2)	22.4	26.8	23.6	9.7
		N(0,3)	9.6	47.8	39.4	21.2
		N(0,4)	3.1	66.4	51.1	36.3
	.05	N(0,2)	45.7	30.0	39.9	9.0
		N(0,3)	24.5	74.0	52.2	29.9
		N(0,4)	10.6	110.7	73.3	52.3
	.01	N(0,2)	117.6	61.9	81.1	20.3
		N(0,3)	96.0	215.7	83.3	39.1
		N(0,4)	53.0	170.9	164.5	86.7

TABLE 9

FOUTZ F_n TEST VS CHI SQUARED TEST
MIXED NORMAL DISTRIBUTIONS

		SIGNIFICANCE LEVEL		
	MIXED NORMAL	.10	.05	.01
n=21	$0.70 \times N(0.5, 1) + 0.30 \times N(0.0, 3)$	24.0	- 0.7	-20.8
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 3)$	14.4	- 9.2	-27.2
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 4)$	17.2	- 3.2	-21.1
n=31	$0.70 \times N(0.5, 1) + 0.30 \times N(0.0, 3)$	9.6	- 6.8	-28.6
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 3)$	3.3	-12.4	-33.3
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 4)$	3.7	-10.7	-30.2
n=51	$0.70 \times N(0.5, 1) + 0.30 \times N(0.0, 3)$	25.1	-24.1	-43.6
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 3)$	- 2.3	-23.6	-48.6
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 4)$	- 4.1	-26.3	-45.5

TABLE 10

FOUTZ F_n TEST VS KOLMOGOROV-SMIRNOV TEST
MIXED NORMAL DISTRIBUTIONS

		SIGNIFICANCE LEVEL		
MIXED NORMAL		.10	.05	.01
n=21	$0.70 \times N(0.5, 1) + 0.30 \times N(0.0, 3)$	-17.0	-28.9	-33.7
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 3)$	-27.3	-38.2	-40.7
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 4)$	-25.5	-32.5	-35.1
n=31	$0.70 \times N(0.5, 1) + 0.30 \times N(0.0, 3)$	-25.4	-34.5	-43.4
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 3)$	-34.8	-45.5	-50.7
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 4)$	-34.6	-40.2	-45.9
n=51	$0.70 \times N(0.5, 1) + 0.30 \times N(0.0, 3)$	-28.7	-47.6	-56.7
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 3)$	-48.7	-57.4	-64.9
	$0.80 \times N(0.5, 1) + 0.20 \times N(0.0, 4)$	-43.3	-51.3	-61.0

TABLE 11

FOUTZ F_n TEST VS CHI SQUARED TEST
PEARSON DISTRIBUTIONS

		SIGNIFICANCE LEVEL			
	β_1	β_2	.10	.05	.01
n=21	0.0	2.30	3.4	-10.5	-20.6
	0.0	2.80	21.1	- 0.9	-12.4
	0.01	1.75	12.8	41.1	57.8
	0.01	1.90	- 9.1	- 2.4	18.9
	0.25	3.20	16.6	- 0.4	- 2.3
	0.50	3.00	17.5	5.8	- 9.3
	1.00	3.40	-12.5	-16.2	-15.8
	1.00	3.60	9.1	-31.4	-30.9
	1.00	3.80	16.1	-17.6	-24.8
	2.00	5.50	-16.1	-14.9	-20.7
n=31	0.0	2.30	- 6.6	-19.0	-22.5
	0.0	2.80	10.7	-12.1	- 5.4
	0.01	1.75	- 5.3	39.4	83.5
	0.01	1.90	-18.7	- 6.9	26.9
	0.25	3.20	0.7	-11.8	- 3.6
	0.50	3.00	- 1.1	- 0.6	-12.2
	1.00	3.40	-25.5	-23.5	-23.4
	1.00	3.60	- 5.7	-41.4	-41.7
	1.00	3.80	0.0	-26.9	-32.9
	2.00	5.50	-29.5	-19.4	-26.5
n=51	0.0	2.30	30.4	-39.0	-19.7
	0.0	2.80	34.9	-40.0	-17.3
	0.01	1.75	6.6	68.1	132.9
	0.01	1.90	-11.8	-15.8	20.7
	0.25	3.20	33.3	-10.8	-15.6
	0.50	3.00	2.5	-19.9	-20.2
	1.00	3.40	-37.4	-35.7	-35.6
	1.00	3.60	-13.6	-53.7	-56.7
	1.00	3.80	- 4.7	-40.6	-46.5
	2.00	5.50	-38.5	-28.7	-38.0

TABLE 12

FOUTZ F_n TEST VS KOLMOGOROV-SMIRNOV TEST
PEARSON DISTRIBUTIONS

			SIGNIFICANCE LEVEL		
	β_1	β_2	.10	.05	.01
n=21	0.0	2.30	-22.8	-25.0	-26.2
	0.0	2.80	5.4	-18.8	- 7.2
	0.01	1.75	- 9.4	20.6	66.0
	0.01	1.90	-33.3	-15.3	16.5
	0.25	3.20	- 8.1	-14.8	- 4.1
	0.50	3.00	9.9	21.7	34.9
	1.00	3.40	84.8	94.4	95.9
	1.00	3.60	52.9	64.5	76.7
	1.00	3.80	38.4	44.0	59.4
	2.00	5.50	62.4	74.7	70.4
n=31	0.0	2.30	-28.9	-36.8	-36.9
	0.0	2.80	11.2	-20.8	-13.0
	0.01	1.75	-14.1	27.6	89.1
	0.01	1.90	-36.7	-15.4	19.4
	0.25	3.20	-12.9	-14.0	- 9.3
	0.50	3.00	- 0.6	25.0	31.3
	1.00	3.40	100.0	134.3	131.7
	1.00	3.60	55.5	79.4	91.8
	1.00	3.80	39.9	46.2	65.3
	2.00	5.50	63.3	91.0	100.0
n=51	0.0	2.30	- 9.1	-48.6	-40.0
	0.0	2.80	- 4.9	-38.9	-35.8
	0.01	1.75	-18.4	62.0	171.8
	0.01	1.90	-37.4	-22.9	26.6
	0.25	3.20	- 8.1	- 9.6	-13.8
	0.50	3.00	0.0	25.9	47.0
	1.00	3.40	169.9	225.8	245.5
	1.00	3.60	64.9	138.9	154.4
	1.00	3.80	42.1	69.0	85.2
	2.00	5.50	103.0	151.0	164.7

5. CONCLUDING REMARKS

The superior performance of the Foutz F_n test in detecting certain types of deviations from the hypothesized distribution leads to several more problems to be considered. Of primary importance is the generation of percentage points for the distribution of F_n for various values of n . The intractability of the problem of obtaining the exact distribution requires an empirical approach to finding a correction to the asymptotic approximation given by Foutz.

Since the test is also applicable to p -variate distributions, an investigation of ways to obtain the statistically equivalent blocks, and then the probability content of them, at least for $p = 2$, is to be considered. The problems of obtaining these blocks and their contents becomes increasingly complicated in higher dimensions.

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THIS PROGRAM GENERATES DEVIATES IN RANDOM STABILIZED STANDARD FORM BLOCKS OF 20, 30, AND 50 DEVIATES ARE CONSIDERED BY GENERATING 50, THEN CONSIDERING THE FIRST 20, THE NEXT 30, AND ALL 50. THE HYPOTHESIS THAT THEY CAME FROM A NORMAL DISTRIBUTION WITH MEAN AND VARIANCE AS GIVEN IN INPUT IS TESTED.

THE TESTS PERFORMED ARE THE CHI SQUARED, KOLMOGOROV-SMIRNOV, AND FOUTZ TEST AT THE CONFIDENCE LEVELS OF 10, 5, AND 1 PERCENT. THE INPUT VALUES ARE MEAN, VARIANCE, ISEED, AND THE NUMBER OF REPLICATIONS.

```

IMPLICIT REAL*8 (A-Z)
INTEGER*4 T, IER, N, I, NP1, N1, N2, N3, ISEED, NR, NSTOP, IDF, IQ, ISEED1
1  , NCEL(3), NST(3), NSMP(3), NCHI(3,3), NCHIN(3,3), NKS(3,3),
2  NKSIN(3,3), NFNN(3,3), NFN(3,3), NSR
DIMENSION RD(55), CFISQU(2),
1  KOLSMR(2), FOUTZL(2), FNTST(3,3), ZALF(3), RN(55)
REAL*4 CELLS(50), CCMP(50), Q, CHI, R(55), PDIF(6), QTST(3), RNN(55),
1  STST(3,3)
DATA STST/.26473,.21756,.16959,.29408,.24170,.18841,.35241,
1  .28987,.22604/
DATA NCEL/8,10,10/, NST/1,21,1/, NSMP/20,30,50/, QTST/.1,.05,.01/
1  , CHISQU/'CHI SQUA', 'RED TEST', KOLSMR/'KOLMOG-S', 'MIR TEST'/
2  , FOUTZL/'FOUTZ TE', 'ST', ZALC1, ZALC2/.243069D0,.367879D0/
DATA WT1, MEAN1, VAR1, WT2, MEAN2, VAR2/.8D0,0.0D0,1.0D0,.2D0,0.0D0,3.0D0/
3  , ZALF/1.28155D0,1.64485D0,2.32635D0/

```

THE NEXT 4 CARDS PUT IN THE EMPIRICAL CRITICAL VALUES FOR THE FOUTZ DISTRIBUTION.

```

                DIMENSION FNEMP(3,3)
1              DATA FNEMP/.42714D0,.41903D0,.40816D0,
2              .44865D0,.43553D0,.42116D0,.48659D0,
                .46579D0,.44487D0/

```

```

REAL*4 ALPHA, BETA, BPRIME
COMMON/NCRPRM/MEAN, VARI, SVARI
EXTERNAL NORM, UNIF
T = 1
PIB2 = DATAN(1.0D0)*2.0D0
100 PRINT 1
READ 2, MEAN, VARI
IF (MEAN+VARI.LE.0.0D0) STOP
2  FORMAT(2E5.0)
SVARI = DSQRT(VARI)
WRITE(4,14)
14  FORMAT(1H1)
1  FORMAT(' + INPUT MEAN AND VARIANCE, FORMAT(2E5.0)')
PRINT 17
READ 12, ALPHA, BETA
17  FORMAT(' + INPUT ALPHA AND BETA, FORMAT(2F5.0)')
12  FORMAT(5F5.0)
PRINT 3
READ 5, ISEED, NR
ISEED1 = ISEED
BPRIME = BETA
IF (ALPHA.EC.0.) GO TO 110
PHIZ = -PIB2*BETA*(1. - ABS(1. - ALPHA))/ALPHA
BPRIME = DTAN(PIB2*(1. - ALPHA))*DTAN(PHIZ*ALPHA)
110 CONTINUE
DSEED = DFLOAT(ISEED)
5  FORMAT(I10,I5)
3  FORMAT(' + INPUT SEED AND NUMBER OF REPLICATIONS, FORMAT(I10,I5)')
DO 205 IQ=1,3
DO 205 N1=1,3
NCHI(N1,IQ) = 0
NCHIN(N1,IQ) = 0
NKS(N1,IQ) = 0
NKSIN(N1,IQ) = 0

```



```

C
C
C
C
NFN(N1,IQ) = 0
FNTST(N1,IQ) = ZALC1*ZALF(IQ)/DSQRT(DFLOAT(NSMP(N1)+1)) + ZALC2
THE FOLLOWING CARD PUTS IN THE EMPIRICAL VALUES FOR THE
  FOUTZ TEST
FNTST(N1,IQ) = FNEMP(N1,IQ)
C
205 NFNN(N1,IQ) = 0
DO 300 I=1,NR
CALL RANSRV(50,RN,RD,DSEED,ALPHA,BPRIME)
DO 250 N1=1,3
N2 = NST(N1)
N3 = NSMP(N1)
CALL VSRTAD(RD(N2),N3)
CALL VSRTAD(RN(N2),N3)
DO 210 NSR=1,N3
RNN(NSR) = RN(NSR + N2-1)
210 R(NSR) = RD(NSR+N2-1)
IDF = 0
CALL GFIT(UNIF,NCEL(N1),RNN,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 241 IC=1,3
IF(Q.LT.OTST(IQ))NCHI(N1,IQ) = NCHI(N1,IQ) + 1
241 CONTINUE
CALL NKS1(UNIF,RNN,N3,PDIF,IER)
DO 242 IC = 1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IQ) = NKS(N1,IQ) + 1
242 CONTINUE
CALL FOUTZ(UNIF,RNN,N3,Q)
DO 243 IC=1,3
IF(Q.GT.FNTST(N1,IQ))NFN(N1,IQ) = NFN(N1,IQ) + 1
243 CONTINUE
IDF = 0
CALL GFIT(NORM,NCEL(N1),R,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 244 IC = 1,3
IF(Q.LT.OTST(IQ))NCHIN(N1,IQ) = NCHIN(N1,IQ) + 1
244 CONTINUE
CALL NKS1(NORM,R,N3,PDIF,IER)
DO 245 IQ=1,3
IF(PDIF(1).GT.STST(N1,IQ))NKSN(N1,IQ) = NKSN(N1,IQ) + 1
245 CONTINUE
CALL FOUTZ(NORM,R,N3,Q)
DO 246 IC=1,3
IF(Q.GT.FNTST(N1,IC))NFNN(N1,IC) = NFNN(N1,IC) + 1
246 CONTINUE
250 CONTINUE
300 CONTINUE
ISEED = DSEED
WRITE(4,7)MEAN,VARI,NR,ISEED1,ISEED
WRITE(4,11)ALPHA,BETA
11 FORMAT(//'RANDOM STABLE STANDARDIZED FORM (RSSF), ALPHA =',F5.2,
1 ' , BETA =',F5.2//)
WRITE(4,8)CHISQU,((NCHI(N1,IQ),
1 N1=1,3),IQ=1,3),((NCHIN(N1,IC),
2 N1=1,3),IQ=1,3)
WRITE(4,8)KOLSMR,((NKS(N1,IQ),N1=1,3),IQ=1,3),
1 ((NKSN(N1,IC),N1=1,3),IQ=1,3)
WRITE(4,8)FOUTZL,((NFN(N1,IQ),N1=1,3),IQ=1,3),((NFNN(N1,IC),N1=1,
1 ),IQ=1,3)
7 FORMAT(//'THE VALUES OF MEAN (U) AND VARIANCE (V) ARE',2F12.5//
9 ' THE RESULTS OF ',I5,' REPLICATIONS, STARTING WITH SEED',
1 I12//33X,'ENDING WITH NEXT SEED',I12//)
8 FORMAT(//20X,2A8//20X,'20 PTS',
2 6X,'30 PTS',6X,'50 PTS'//10X,'10%',3I12/' CONTROL',2X,'5%',
3 3I12/10X,'1%',3I12//)' RSSF ',2X,'10%',3I12/' VS',5X,'5%',
4 3I12/' N (U,V)',2X,'1%',3I12)
WRITE(4,9)((FNTST(N1,IQ),N1=1,3),IC=1,3)
9 FORMAT(/'O THIS WAS BASED ON THE FOLLOWING TEST VALUES',
1 /(13X,3F12.5))
GO TO 100

```


END

```
200 SUBROUTINE RANSRV(ND,RAN,R,DSEED,ALPHA,BPRIME)
      IMPLICIT REAL*8 (A-M,O-Z)
      DIMENSION R(1),RAN(1)
      REAL*4 RN(100),ALPHA,BPRIME,RSTAB,W
      CALL GSUBS(DSEED,2*ND,RN)
      DO 200 NA=1,ND
        RAN(NN) = RN(NN)
        W = -DLOG(1.DO - RN(NN+ND))
        R(NN) = RSTAB(ALPHA,BPRIME,RN(NN),W)
      CONTINUE
      RETURN
      END
```

```
SUBROUTINE NCRM(X,P)
      REAL*8 MEAN,VARI,SVARI
      COMMON/NCRPRM/MEAN,VARI,SVARI
      T = (X - MEAN)/SVARI
      P = .5*ERFC(-T*.7071068)
      RETURN
      END
```

```
SUBROUTINE UNIF(X,P)
      P = X
      RETURN
      END
```

```
FUNCTION RSTAB(ALPHA,BPRIME,U,W)
      RSTAB IS A RANDOM STABLE STANDARDIZED FORM (WHATEVER)
```

```
      ARGUMENTS
        ALPHA = CHARACTERISTIC EXPONENT
        BPRIME = SKEWNESS IN REVISED PARAMETERIZATION
        U = UNIFORM VARIATE ON (0,1)
        W = EXPONENTIALLY DISTRIBUTED VARIATE
      DOUBLE PRECISION CA,CB
      DATA PIBY2/1.57079633/,PIBY4/.785398163/,THR1/.99/
      EPS = 1. - ALPHA
      COMPUTE SOME TANGENTS
      PHIBY2 = PIBY2*(U - .5)
      A = PHIBY2*TAN2(PHIBY2)
      BB = TAN2(EPS*PHIBY2)
      B = EPS*PHIBY2*BB
      IF(EPS.GT.-.99)TAU = BPRIME/(TAN2(EPS*PIBY2)*PIBY2)
      IF(EPS.LE.-.99)TAL = BPRIME*PIBY2*EPS*(1. - EPS)*TAN2((1. - EPS)*
1    PIBY2)
      COMPUTE SOME NECESSARY SUBEXPRESSIONS
      IF PHI NEAR PI BY 2, USE DOUBLE PRECISION.
      IF(A.GT.THR1)GO TO 50
      SINGLE PRECISION
      A2 = A**2
      A2P = 1. + A2
      A2 = 1. - A2
      B2 = B**2
      B2P = 1. + B2
      B2 = 1. - B2
      GO TO 100
      DOUBLE PRECISION
50    DA = DBLE(A)**2
      DB = DBLE(B)**2
      A2 = 1.DO - DA
      A2P = 1.DO + DA
      B2 = 1. - DB
      B2P = 1. + DB
      COMPUTE COEFFICIENT
```

```

100 Z = A2P*(B2 + 2.*PHIBY2*BB*TAU)/(W*A2*B2P)
C   COMPUTE THE EXPONENTIAL-TYPE EXPRESSION
    ALOGZ = ALOG(Z)
    D = D2(EPS*ALOGZ/(1. - EPS))*(ALOGZ/(1. - EPS))
C   COMPUTE STABLE
    RSTAB = (1. + EPS*D)*2.*((A - B)*(1. + A*B) - PHIBY2*TAU*BB*(B*A2
1 - 2.*A))/(A2*B2P) + TAU*D
    RETURN
END

```

```

FUNCTION D2(Z)
C   EVALUATE (EXP(X) - 1)/X
    DOUBLE PRECISION F1,F2,Q1,Q2,Q3,PV,ZZ
    DATA P1,P2,Q1,Q2,Q3/.840066852536483239D3,.200011141589964569D2,
1 .168013370507926648D4,.180013370407390023D3,1.D0/
C   THE APPROXIMATION 1801 FROM HART ET AL (1968,P213)
    IF(ABS(Z).GT.0.1)GO TO 100
    ZZ = Z*Z
    PV = P1 + ZZ*P2
    C2 = 2.*C0*PV/(Q1 + ZZ*(Q2 + ZZ*Q3) - Z*PV)
    RETURN
100 D2 = (EXP(Z) - 1.)/Z
    RETURN
END

```

```

FUNCTION TAN(XARG)
C   TANGENT FUNCTION
    LOGICAL NEG,INV
    DATA P0,P1,P2,Q0,Q1,Q2/.129221035E3,-.887662377E1,.528644456E-1,
1 .164529332E3,-.451320561E2,1./
C   THE APPROXIMATION 4283 FROM HART ET AL (1968, P. 251)
    DATA PIBY4 /.785398163/,PIBY2/1.57079633/,PI/3.14159265/
    NEG = .FALSE.
    INV = .FALSE.
    X = XARG
    NEG = X.LT.0.
    X = ABS(X)
C   PERFORM RANGE REDUCTION IF NECESSARY
    IF(X.LE.PIBY4)GO TC 50
    X = AMOD(X,PI)
    IF(X.LE.PIBY2)GO TO 30
    NEG = .NOT.NEG
    X = PI - X
30 IF(X.LE.PIBY4) GC TC 50
    INV = .TRUE.
    X = PIBY2 - X
50 X = X/PIBY4
C   CONVERT TO RANGE CF RATIONAL
    XX = X*X
    TAN = X*(P0 + XX*(P1 + XX*P2))/(Q0 + XX*(Q1 + XX*Q2))
    IF(NEG)TAN = -TAN
    IF(INV)TAN = 1./TAN
    RETURN
END

```

```

FUNCTION TAN2(XARG)
C   COMPUTE TAN(X)/X
C   FUNCTION DEFINED ONLY FOR ABS(XARG).LE.PI BY 4
C   FOR OTHER ARGUMENT RETURNS TAN(X)/X, COMPUTED DIRECTLY
    DATA P0,P1,P2,Q0,Q1,Q2/.129221035E3,-.887662377E1,.528644456E-1,
1 .164529332E3,-.451320561E2,1./
C   THE APPROXIMATION 4283 FROM HART ET AL (1968, P. 251)
    DATA PIBY4,PIBY2,PI/.785398163,1.57079633,3.14159265/
    X = ABS(XARG)
    IF(X.GT.PIBY4)GC TC 200
    X = X/PIBY4
C   CONVERT TO RANGE CF RATIONAL

```

THIS PROGRAM GENERATES MIXED NORMAL DEVIATES $WT1*N(MEAN1,VAR1) + (1 - WT1)*N(MEAN2,VAR2)$.
 BLOCKS OF 20, 30, AND 50 DEVIATES ARE CONSIDERED BY GENERATING 50. THEN CONSIDERING THE FIRST 20, THE NEXT 30, AND ALL 50. THE HYPOTHESIS THAT THEY CAME FROM A NORMAL DISTRIBUTION WITH MEAN AND VARIANCE AS GIVEN IN INPUT IS TESTED.

THE TESTS PERFORMED ARE THE CHI SQUARED, KOLMOGOROV-SMIRNOV, AND FOUTZ TEST AT THE CONFIDENCE LEVELS OF 10, 5, AND 1 PERCENT. THE INPUT VALUES ARE MEAN, VARIANCE, ISEED, AND THE NUMBER OF REPLICATIONS.

```

IMPLICIT REAL*8 (A-Z)
INTEGER*4 T,IER,N,I,NP1,N1,N2,N3,ISEED,NR,NSTOP,IOF,IQ,ISEED1
1  ,NCEL(3),NST(3),NSMP(3),NCHI(3,3),NCHIN(3,3),NKS(3,3),
2  ,NKSIN(3,3),NFNN(3,3),NFN(3,3),NSR
DIMENSION RD(55),CHISQU(2),
1  KOLSMR(2),FOUTZL(2),FNTST(3,3),ZALF(3),RN(55)
REAL*4 CELLS(50),CCMP(50),Q,CFI,R(55),POIF(6),QTST(3),RNN(55),
1  STST(3,3)
DATA STST/.26473,.21756,.16959,.29408,.24170,.18841,.35241,
1  .28987,.2264/
DATA NCEL/8,10,10/,NST/1,21,1/,NSMP/20,30,50/,QTST/.1,.05,.01/
1  ,CHISQU/'CHI SQUA','RED TEST',KOLSMR/'KOLMOG-S','MIR TEST'/
2  ,FOUTZL/'FOUTZ TE','ST'/,ZALC1,ZALC2/.243069D0,.367879D0/
DATA WT1,MEAN1,VAR1,WT2,MEAN2,VAR2/.8D0,0.00,1.0D0,.2D0,0.00,3.0D0/
3  ,ZALF/1.28155D0,1.64485D0,2.32635D0/

```

THE NEXT 4 CARDS PUT IN THE EMPIRICAL CRITICAL VALUES FOR THE FOUTZ TEST.

```

DIMENSION FNEMP(3,3)
DATA FNEMP/.42714D0,.41903D0,.40816D0,
.44865D0,.43553D0,.42116D0,.48659D0,
.46579D0,.44487D0/

```

```

COMMON/NCRPRM/MEAN,VARI,SVARI
EXTERNAL NORM,UNIF
T = 1
100 PRINT 1
READ 2,MEAN,VARI
IF (MEAN+VARI.LE.0.D0) STOP
2  FORMAT(2E5.0)
SVARI = CSQRT(VARI)
WRITE(4,14)
14  FORMAT(1F1)
1  FORMAT('INPUT MEAN AND VARIANCE, FORMAT(2E5.0)')
PRINT 17
READ 12,WT1,MEAN1,VAR1,MEAN2,VAR2
WT2 = 1.00 - WT1
17  FORMAT('INPUT WT1,MEAN1,VAR1,MEAN2,VAR2, FORMAT(5F5.0)')
12  FORMAT(5F5.0)
PRINT 3
READ 5,ISEED,NR
ISEED1 = ISEED
DSEED = CFLOAT(ISEED)
5  FORMAT(I10,I5)
3  FORMAT('INPUT SEED AND NUMBER OF REPLICATIONS, FORMAT(I10,I5)')
DO 205 IQ=1,3
DO 205 N1=1,3
NCHI(N1,IQ) = 0
NCHIN(N1,IQ) = 0
NKS(N1,IQ) = 0
NKSIN(N1,IQ) = 0
NFN(N1,IQ) = 0
FNTST(N1,IQ) = ZALC1*ZALF(IQ)/DSQRT(DFLOAT(NSMP(N1)+1)) + ZALC2
FNTST(N1,IQ) = FNEMP(N1,IQ)
205 NFNN(N1,IQ) = 0
DO 300 I=1,NR

```

```

CALL MIXNRM(50,RN,RD,DSEED,WT1,MEAN1,VAR1,MEAN2,VAR2)
DO 250 N1=1,3
N2 = NST(N1)
N3 = NSMP(N1)
CALL VSRTAD(RD(N2),N3)
CALL VSRTAD(RN(N2),N3)
DO 210 NSR=1,N3
RNN(NSR) = RN(NSR + N2-1)
210 R(NSR) = RD(NSR+N2-1)
IDF = 0
CALL GFIT(UNIF,NCEL(N1),RNN,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 241 IQ=1,3
IF(Q.LT. CTST(IQ))NCHI(N1,IQ) = NCHI(N1,IQ) + 1
241 CONTINUE
CALL NKS1(UNIF,RNN,N3,PDIF,IER)
DO 242 IC = 1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IQ) = NKS(N1,IQ) + 1
242 CONTINUE
CALL FOUTZ(UNIF,RNN,N3,Q)
DO 243 IQ=1,3
IF(Q.GT. FNTST(N1,IC))NFN(N1,IQ) = NFN(N1,IQ) + 1
243 CONTINUE
IDF = 0
CALL GFIT(NORM,NCEL(N1),R,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 244 IC = 1,3
IF(Q.LT. CTST(IQ))NCHIN(N1,IQ) = NCHIN(N1,IQ) + 1
244 CONTINUE
CALL NKS1(NORM,R,N3,PDIF,IER)
DO 245 IQ=1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IQ) = NKS(N1,IQ) + 1
245 CONTINUE
CALL FOUTZ(NORM,R,N3,Q)
DO 246 IC=1,3
IF(Q.GT. FNTST(N1,IC))NFNN(N1,IQ) = NFNN(N1,IQ) + 1
246 CONTINUE
250 CONTINUE
300 CONTINUE
ISEED = CSEED
WRITE(4,7)MEAN,VAR1,RN,ISEED1,ISEED
WRITE(4,11)WT1,MEAN1,VAR1,WT2,MEAN2,VAR2
11 FORMAT(// ' MIXED NORMAL (MIXNCRM) = ',F5.2,' *N(',F3.1,',',F3.1,
1 ' ) + ',F5.2,' *N(',F3.1,',',F3.1,')' //)
WRITE(4,8)CHISCU,((NCHI(N1,IQ),
1 N1=1,3),IQ=1,3),((NCHIN(N1,IC),
2 N1=1,3),IC=1,3)
WRITE(4,8)KOLSMR,((NKS(N1,IQ),N1=1,3),IQ=1,3),
1 ((NKS(N1,IQ),N1=1,3),IC=1,3)
WRITE(4,8)FOUTZL,((NFN(N1,IQ),N1=1,3),IQ=1,3),((NFNN(N1,IQ),N1=1
1 ),IQ=1,3)
7 FORMAT(// ' THE VALUES OF MEAN (U) AND VARIANCE (V) ARE',2F12.5//
9 ' THE RESULTS OF ',I5,' REPLICATIONS, STARTING WITH SEED',
1 I12//33X,'ENDING WITH NEXT SEED',I12//)
8 FORMAT(///20X,2A8//20X,'20 PTS',
2 6X,'30 PTS',6X,'50 PTS'//10X,'10%',3I12/' CONTROL',2X,' 5%',
3 3I12/10X,' 1%',3I12// ' MIXNCRM',2X,'10%',3I12/' VS',5X,' 5%',
4 3I12/' N (U,V)',2X,' 1%',3I12)
WRITE(4,9)((FNTST(N1,IQ),N1=1,3),IC=1,3)
9 FORMAT(// '0 THIS WAS BASED ON THE FOLLOWING TEST VALUES',
1 /(13X,3F12.5))
GO TO 100
END

```

```

SUBROUTINE MIXNRM(ND,RAN,R,DSEED,WT1,MEAN1,VAR1,MEAN2,VAR2)
IMPLICIT REAL*8 (A-M,C-Z)
DIMENSION R(1),RAN(1)
REAL*4 RN(100),RNN,RNNN
SVAR1 = DSQRT(VAR1)
SVAR2 = DSQRT(VAR2)
WT2 = 1.00 - WT1

```



```

XX = X*X
TAN2 = (P0 + XX*(P1 + XX*P2))/(PIBY4*(Q0 + XX*(Q1 + XX*Q2)))
RETURN
200 TAN2 = TAN(XARG)/XARG
RETURN
END

```

```

SUBROUTINE FCUTZ (PCDF,XT,NXT,FN)

```

THIS SUBROUTINE GENERATES THE STATISTIC FOR THE FCUTZ FN TEST.

INPUT VARIABLES ARE:

PCDF - THE CUMULATIVE DISTRIBUTION FUNCTION AGAINST WHICH THE
DEVIATES ARE BEING TESTED. CALLING SEQUENCE MUST BE OF
THE FORM 'CALL PCDF(X,P)', WHERE X IS AN INPUT VALUE,
AND THE VALUE OF THE CUMULATIVE DISTRIBUTION FUNCTION
IS RETURNED IN P.
P MUST BE BETWEEN 0 AND 1.

XT - THE ARRAY OF DEVIATES, IN INCREASING ORDER.

NXT - THE NUMBER OF DEVIATES IN THE ARRAY XT (= N - 1)

THE RETURNED VALLE IS FN, THE VALUE OF THE STATISTIC.

NXT IS PRESENTLY LIMITED TO A MAXIMUM OF 50 BY THE DIMENSION OF
THE VARIABLE XTD.

```

DIMENSION XT(1)
REAL*8 XTD(51),RN,FND

```

```

N = NXT + 1
DO 200 I=1,NXT

```

```

K = N - I

```

```

CALL PCDF(XT(K),P)

```

```

200 XTD(K+1) = P

```

```

RN = 1.00/N

```

```

XTD(1) = RN - XTD(2)

```

```

DO 300 I=2,NXT

```

```

300 XTD(I) = RN - XTD(I+1) + XTD(I)

```

```

XTD(N) = RN - 1.00 + XTD(N)

```

```

FND = 0.

```

```

DO 400 I=1,N

```

```

400 FND = FND + CMAX1(XTD(I),0.00)

```

```

FN = FND

```

```

RETURN

```

```

END

```

```

CALL GGUBS(DSEED,2*ND,RN)
DO 200 NN=1,ND
RAN(NN) = RN(NN)
CALL MDNRIS(RN(NN),RNN,IER)
IF(RN(NN+ND).GT.WT2)GO TO 150
R(NN) = RNN*SVAR2 + MEAN2
GO TO 200
150 R(NN) = RNN*SVAR1 + MEAN1
200 CONTINUE
RETURN
END

```

```

SUBROUTINE NCRM(X,P)
REAL*8 MEAN,VAR1,SVARI
COMMON/NCRPRM/MEAN,VAR1,SVARI
T = (X - MEAN)/SVARI
P = .5*ERFC(-T*.7071068)
RETURN
END

```

```

SUBROUTINE UNIF(X,P)
P = X
RETURN
END

```

```

SUBROUTINE FOUTZ (PCDF,XT,NXT,FN)
THIS SUBROUTINE GENERATES THE STATISTIC FOR THE FOUTZ FN TEST.
INPUT VARIABLES ARE:
  PCDF - THE CUMULATIVE DISTRIBUTION FUNCTION AGAINST WHICH THE
        DEVIATES ARE BEING TESTED. CALLING SEQUENCE MUST BE
        THE FORM 'CALL PCDF(X,P)', WHERE X IS AN INPUT VALUE
        AND THE VALUE OF THE CUMULATIVE DISTRIBUTION FUNCTION
        IS RETURNED IN P.
        P MUST BE BETWEEN 0 AND 1.
  XT   - THE ARRAY OF DEVIATES, IN INCREASING ORDER.
  NXT  - THE NUMBER OF DEVIATES IN THE ARRAY XT   (= N - 1)

```

THE RETURNED VALUE IS FN, THE VALUE OF THE STATISTIC.

```

NXT IS PRESENTLY LIMITED TO A MAXIMUM OF 50 BY THE DIMENSION OF
THE VARIABLE XTD.
DIMENSION XT(1)
REAL*8 XTD(51),RN,FND
N = NXT + 1
DO 200 I=1,NXT
K = N - I
CALL PCDF(XT(K),P)
200 XTD(K+1) = P
RN = 1.00/N
XTD(1) = RN - XTD(2)
DO 300 I=2,NXT
300 XTD(I) = RN - XTD(I+1) + XTD(I)
XTD(N) = RN - 1.00 + XTD(N)
FND = 0.
DO 400 I=1,N
400 FND = FND + CMAX1(XTD(I),0.00)
FN = FND
RETURN
END

```

```

THIS PROGRAM GENERATES PEARSON TYPE I OR II RANDOM DEVIATES .
THE PARAMETERS ARE CALCULATED IN TERMS OF B1 AND B2 IN
SUBROUTINE PRM. SUBROUTINE ADINT1 IS USED TO CALCULATE THE
CDF TABLE. GGUBS IS USED TO GENERATE RANDOM (0,1) DEVIATES
(UNIFORMLY DISTRIBUTED, AND THEN SUBROUTINE RANDP1 DOES
AND INVERSE CALCULATION TO OBTAIN THE RANDOM
PEARSON DEVIATE.)
A SET OF 50 RANDOM DEVIATES ARE GENERATED, THEN THE CHI
SQUARED TEST, THE KOLMOGOROV-SMIRNOV TEST, AND THE FOUTZ FN
TEST ARE APPLIED. THE UNIFORM DEVIATES ARE TESTED AGAINST
THE HYPOTHESIS THEY CAME FROM A UNIFORM DISTRIBUTION AS A CONTROL.
THEN THE PEARSON DEVIATES ARE TESTED AGAINST THE HYPOTHESIS
THAT THEY CAME FROM A NORMAL DISTRIBUTION.
THIS TEST IS REPLICATED A NUMBER OF TIMES.

```

THE TESTS ARE APPLIED TO THE FIRST 20, THE NEXT 30, AND ALL 50.

THE SET OF DEVIATES IS ALSO TESTED AGAINST THE NORMAL DISTRIBUTION
THIS IS REPEATED FOR THE SAME SETS.

THE INPUT VALUES ARE B1,B2,ISEED, AND THE NUMBER OF REPLICATIONS

```

      IMPLICIT REAL*8 (A-Z)
      INTEGER*4 T, IER, N, I, NP1, N1, N2, N3, ISEED, NR, NSTOP, IDF, IQ, ISEED1
1    NCEL(3), NST(3), NSMP(3), NCHI(3,3), NCHIN(3,3), NKS(3,3),
2    NKSIN(3,3), NFNN(3,3), NFN(3,3), NAFNN(3,3), NAFN(3,3), NSR
      DIMENSION X(2001), CDF(2001), A(2001), B(2001), RD(55), CHISQU(2),
1    KOLSMR(2), FOUTZL(2), ASFL(2), FNTST(3,3), ZALF(3), RN(55)
      COMMON /PPARM/CO, C1, C2, A1, A2, BCO, BC1, M1, M2, KINV, XL, XR, MEAN, T
      COMMON /CDF/X, CDF, A, B, NP1
      REAL*4 CELLS(50), CCMP(50), Q, CHI, R(55), PDIF(6), QTST(3), RNN(55),
1    STST(3,3)
      DATA STST/.26473,.21756,.16959,.29408,.24170,.18841,.35241,
1    .28987,.22604/
      DATA NCEL/8,10,10/, NST/1,21,1/, NSMP/20,30,50/, QTST/.1,.05,.01/
1    , CHISQU/'CHI SQUA', 'RED TEST'/, KOLSMR/'KOLMOG-S', 'MIR TEST'/'
2    , FOUTZL/'EMP FOUT', 'Z TEST'/, ZALC1, ZALC2/.243069CO, .367879DO/
3    , ZALF/1.28155CO, 1.64485DO, 2.32635CO/
      DIMENSION FNEMP(3,3)
      DATA FNEMP/.42714DO, .41903DO, .40816DO,
2    .44865DO, .43553DO, .42116DO,
3    .48659DO, .46579DO, .44487DO/
4    , ASFL/'ASYM FOU', 'TZ TEST'/'
      EXTERNAL NCRM, UNIF
      T = 1
100 PRINT 1
      READ 2, B1, B2
      IF(B1+B2.LE.0.DO) STOP
2    FORMAT(2E5.0)
      WRITE(4,14)
14    FORMAT(1H1)
1    FORMAT(' + B1 AND B2, FORMAT(2E5.0)')
      CALL PRN(B1, B2, 1.CO, IER)
      IF(IER.NE.0) GO TO 900
      A(1) = 0.DO
      CDF(1) = 0.
      CALL INTSZ(N, X)
      NP1 = N + 1
      PRINT 3
      READ 5, ISEED, NR
      ISEED1 = ISEED
      OSEED = CFLOAT(ISEED)
5    FORMAT(I10,I5)
3    FORMAT(' + INPUT SEED AND NUMBER OF REPLICATIONS, FORMAT(I10,I5)')
      DO 205 IQ=1,3
      DO 205 N1=1,3
      NCHI(N1,IQ) = 0
      NCHIN(N1,IQ) = 0
      NKS(N1,IQ) = 0

```

```

NКСN(N1,IQ) = 0
NFN(N1,IQ) = 0
FNTST(N1,IQ) = ZALC1*ZALF(IQ)/DSQRT(DFLOAT(NSMP(N1)+1)) + ZALC2
NAFNN(N1,IQ) = 0
NAFN(N1,IQ) = 0
205 NFNN(N1,IQ) = 0
DO 300 I=1,NR
CALL RANPD1(50,RN,RC,DSEED)
DO 250 N1=1,3
N2 = NST(N1)
N3 = NSMP(N1)
CALL VSRAD(RD(N2),N3)
CALL VSRAD(RN(N2),N3)
DO 210 NSR=1,N3
RNN(NSR) = RN(NSR + N2-1)
210 R(NSR) = RD(NSR+N2-1)
IDF = 0
CALL GFIT(UNIF,NCEL(N1),RNN,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 241 IQ=1,3
IF(Q.LT.OTST(IQ))NCHI(N1,IQ) = NCHI(N1,IQ) + 1
241 CONTINUE
CALL NKС1(UNIF,RN,N3,PDIF,IER)
DO 242 IQ = 1,3
IF(PDIF(1).GT.STST(N1,IQ))NKS(N1,IQ) = NKS(N1,IQ) + 1
242 CONTINUE
CALL FOUTZ(UNIF,RNN,N3,Q)
DO 243 IQ=1,3
IF(Q.GT.FNEMP(N1,IQ))NFN(N1,IQ) = NFN(N1,IQ) + 1
IF(Q.GT.FNTST(N1,IQ))NAFN(N1,IQ) = NAFN(N1,IQ) + 1
243 CONTINUE
IDF = 0
CALL GFIT(NORM,NCEL(N1),R,N3,CELLS,COMP,CHI,IDF,Q,IER)
DO 244 IQ = 1,3
IF(Q.LT.OTST(IQ))NCHIN(N1,IQ) = NCHIN(N1,IQ) + 1
244 CONTINUE
CALL NKС1(NORM,R,N3,PDIF,IER)
DO 245 IQ=1,3
IF(PDIF(1).GT.STST(N1,IQ))NКСN(N1,IQ) = NКСN(N1,IQ) + 1
245 CONTINUE
CALL FOUTZ(NORM,R,N3,Q)
DO 246 IQ=1,3
IF(Q.GT.FNEMP(N1,IQ))NFNN(N1,IQ) = NFNN(N1,IQ) + 1
IF(Q.GT.FNTST(N1,IQ))NAFNN(N1,IQ) = NAFNN(N1,IQ) + 1
246 CONTINUE
250 CONTINUE
300 CONTINUE
ISEED = DSEED
WRITE(4,7)B1,B2,A1,A2,M1,M2,NR,ISEED1,ISEED
WRITE(4,8)CHISQU,((NCHI(N1,IQ),
1 N1=1,3),IQ=1,3),((NCHIN(N1,IQ),
2 N1=1,3),IQ=1,3)
WRITE(4,8)KCLSMR,((NKS(N1,IQ),N1=1,3),IQ=1,3),
1 ((NКСN(N1,IQ),N1=1,3),IQ=1,3)
WRITE(4,8)FOUTZL,((NFN(N1,IQ),N1=1,3),IQ=1,3),((NFNN(N1,IQ),N1=1,3),
1 IQ=1,3)
WRITE(4,8)ASFL,((NAFN(N1,IQ),N1=1,3),IQ=1,3),
1 ((NAFNN(N1,IQ),N1=1,3),IQ=1,3)
7 FORMAT(// ' THE VALUES OF B1 AND B2 ARE',2F12.5///
A ' VALUES OF A1, A2, M1, AND M2 ARE'//4F12.6///
9 ' THE RESULTS OF ',I5,' REPLICATIONS, STARTING WITH SEED',
1 I12//33X,' ENDING WITH NEXT SEED',I12//)
8 FORMAT(///20X,2A8//20X,' 20 PTS',
2 6X,' 30 PTS',6X,' 50 PTS'//10X,' 10%',3I12/' CONTROL',2X,' 5%',
3 3I12//10X,' 1%',3I12///' PEARSON',2X,' 10%',3I12/' VS',5X,' 5%',
4 3I12/' NORMAL',3X,' 1%',3I12)
GO TO 100
900 PRINT 4,N,IER,B1,B2,ERROR
4 FORMAT('N,IER,B1,B2',2I10,1P3C15.6)
STOP
END

```



```

SUBROUTINE RANPD1(NC,RAN,R,DSEED)
IMPLICIT REAL*8 (A-M,C-Z)
DIMENSION R(1),RAN(1)
REAL*4 RN(100),RNN
CALL GGUBS(DSEED,ND,RN)
DO 200 NN=1,ND
RAN(NN) = RN(NN)
CALL CDFINV(RAN(NN),RNN)
R(NN) = RNN
200 CONTINUE
RETURN
END

```

```

SUBROUTINE NORM(X,P)
P = .5*ERFC(-X*.7071068)
RETURN
END

```

```

SUBROUTINE UNIF(X,P)
P = X
RETURN
END

```

```

SUBROUTINE PRM(B1,B2,U2,IER)
IMPLICIT REAL*8 (A-Z)
COMMON /PPARM/CO,C1,C2,A1,A2,BIGCO,BIGC1,M1,M2,KINV,XL,XR,MEAN,T
INTEGER*4 T,IER
IER = 0
IF(T.EQ.3)B2 = (6. + 3.*B1)/2.
IF(T.EQ.5)B1 = 4.*(2.*B2 - 6.)*(4.*B2-3.)/((B2+3.)**2+
1 12.*(4.*B2-3.))
DEN = 10.*B2 - 12.*B1 - 18.
CO = (4.*B2 - 3.*B1)*U2/DEN
C1 = DSQRT(U2*B1)*(B2 + 3.)/DEN
C2 = (2.*B2 - 3.*B1 - 6.)/DEN
A1 = 0.
A2 = 0.
BIGCO = 0.
BIGC1 = 0.
M1 = 0.
M2 = 0.
GO TO (100,200,300,400,500,600,700),T
100 CALL RCCTS(CO,C1,C2,A1,A2,IER)
IF(IER.NE.0)RETURN
M1 = (C1 + A1)/C2/(A2 - A1)
M2 = -(C1 + A2)/C2/(A2 - A1)
KINV = (A2 - A1)**(M1+ M2 + 1.)*DGAMMA(M1+1.)*DGAMMA(M2+1.)
1 /DGAMMA(M1 + M2 + 2.)
XL = A1
XR = A2
GO TO 750
200 GO TO 100
300 M1 = (CO/C1 - C1)/C1
C2 = 0.
KINV = DGAMMA(M1+1.)*DEXP(CO/C1**2)*DABS(C1)**(2.*M1+1.)
XL = -CO/C1
XR = 1.D50
GO TO 750
400 BIGCO = CO - C1**2/C2/4.
BIGC1 = C1/C2/2.
M1 = (C1 - BIGC1)/DSQRT(2.D0)/BIGCO
M2 = DSQRT(BIGCO/C2)
KINV = 1.D0
XL = -1.D50
XR = 1.D50
GO TO 750
500 BIGC1 = C1/2./C2

```

```

M1 = (C1 - BIGC1)/C2
IF(C2.GT.1.D0)GO TO 800
KINV = CABS(M1)**(1./C2 - 1.)/DGAMMA(1./C2-1.)
XL = -BIGC1
XR = 1.D50
GO TO 750
600 CALL ROOTS(C0,C1,C2,A1,A2,IER)
M1 = -(C1 + A1)/C2/(A2 - A1)
M2 = (C1 + A2)/C2/(A2 - A1)
IF(M2.GE.-1.D0.OR.M1 + M2.GE.0.D0)GO TO 800
KINV = (A2 - A1)**(M1 + M2 + 1.)*DGAMMA(M2+1.)*DGAMMA(-M1-M2-1.)
1 /DGAMMA(-M1)
XL = A2
XR = 1.D50
GO TO 750
700 KINV=C0**(-.5D0/C2)*DSQRT(C0/C2)*DGAMMA(.5D0)*DGAMMA(.5D0/C2-.5D
1 /DGAMMA(.5D0/C2)
C1 = 0.
XL = -1.D50
XR = 1.C50
750 MEAN = (4.D0*B2 - 3.D0*B1)/3.D0/(B2 - B1 - 1.D0)*U2*PDF(0.D0)
RETURN
800 IER = 1
RETURN
END

```

```

SUBROUTINE ROOTS(C0,C1,C2,A1,A2,IER)
IMPLICIT REAL*8 (A-Z)
INTEGER*4 IER
IER = 0
IF(C2.EQ.0.D0)GC TC 500
DIS = C1**2 - 4.CC*C0*C2
IF(DIS.LT.0.D0)GC TC 600
SDIS = DSQRT(DIS)
DNUM = -C1 - SDIS
IF(C1.LT.0.D0)DNUM = -C1 + SDIS
X2 = DNUM/2.D0/C2
X1 = C0/C2/X2
A1 = DMIN1(X1,X2)
A2 = DMAX1(X1,X2)
RETURN
500 A2 = 1.D75
A1 = -C0/C1
RETURN
600 IER = -1
RETURN
END

```

FUNCTION PDF(X)

THIS FUNCTION EVALUATES THE PEARSON DISTRIBUTION
FOR A GIVEN X, THE PARAMETERS HAVING PREVIOUSLY BEEN
CALCULATED IN SUBROUTINE PRM.

```

C
C
C
IMPLICIT REAL*8 (A-Z)
INTEGER*4 IY
COMMON/PPARM/ C0,C1,C2,A1,A2,BIGC0,BIGC1,M1,M2,KINV,X1,X2,MEAN,I
GO TO (100,200,300,400,500,600,700),IY
100 IF(X.LE.A1.OR.X.GE.A2)GO TO 140
PDF = (X - A1)**M1*(A2 - X)**M2/KINV
RETURN
140 IF(X.LE.A1.AND.M1.LT.0.D0)GO TO 150
IF(X.GE.A2.AND.M2.LT.0.D0)GO TO 150
PDF = 1.D-25
RETURN
150 PDF = 1.C25
RETURN
200 GO TO 100

```

```

300 PDF = (C0 + C1*X)**M1*DEXP(-X/C1)/KINV
RETURN
400 PDF = (BIGC0 + C2*(X+BIGC1)**2)**(-1./C2)*
1 DEXP(-M1*DATAN((X + BIGC1)/M2))/KINV
RETURN
500 PDF = (X + BIGC1)**(-1./C2)*DEXP(M1/(X + BIGC1))/KINV
RETURN
600 PDF = (X - A1)**M1*(X - A2)**M2/KINV
RETURN
700 PDF = (CC + C2*X**2)**(-.5D0/C2)/KINV
RETURN
END

```

```

SUBROUTINE INTSZ(N,X)
IMPLICIT REAL*8 (A-Z)
COMMON /FPARM/CO,C1,C2,A1,A2,BC0,BC1,M1,M2,KINV,XL,XR,MEAN,T
COMMON /CDF/XDUM(2001),CDF(2001),A(2001),B(2001),NP1
DIMENSION X(1)
INTEGER*4 T,N,NN,INT,IEND,NP1,IER
X(1) = A1
B(1) = PCF(A1)
CDF(1) = 0.00
INT = 1
NN = 1
DXM = (A2 - A1)/1.07
CX = DXM*1.03
GO TO 160
150 DX = DX*(1.00 + DSQRT(.475D-4/EREST))/2.
INT = 1
160 X(NN+1) = X(NN) + CX
IEND = 1
IF(X(NN+1).LT.A2)GO TO 170
X(NN+1) = A2
DX = X(NN+1) - X(NN)
IEND = 2
170 B(NN+1) = PDF(X(NN+1))
ESTINT = (B(NN+1) + B(NN))*DX/2.00
IF(ESTINT.LT.1.0-8)GO TO 301
300 DCDF = ADINT1(X(NN),X(NN+1),ERRCR,IER)
IF(IER.GT.100)GO TO 900
IF(DCDF.GT..025D0)GO TO 309
EREST = (1.00/B(NN+1) - 1.00/B(NN))*DCDF/8.00
A(NN) = EREST
EREST = CABS(EREST)
GO TO (302,308),INT
301 DCDF = ESTINT
EREST = 5.27777777777D-6
A(NN) = EREST
GO TO 308
302 IF(EREST.LE..5D-4)GO TO 308
CX = DX*(DSQRT(.25D-4/EREST) + 1.00)*.500
IF(DX.GT.DXM)GO TO 304
DX = DXM
X(NN+1) = X(NN) + CX
INT = 2
GO TO 170
304 X(NN+1) = X(NN) + CX
IEND = 1
GO TO 170
308 CDF(NN+1) = CDF(NN) + DCDF
NN = NN + 1
IF(NN.GT.2000)GO TO 910
GO TO (150,310),IEND
309 DX = DX*.0125D0/DCDF
GO TO 160
310 N = NN - 1
NP1 = NN
DO 400 NA=1,N
400 CDF(NN+1) = CDF(NN+1)/CDF(N+1)

```

```

2 FORMAT(' CDF(NN) = ',F20.15)
RETURN
900 PRINT 1,NN,X(NN),X(NN+1),ERRCR
STOP
1 FORMAT(' I,XI,XI+1,ERRCR',I5,1P3D15.6)
910 PRINT 4,X(NN)
4 FORMAT(' RUN OUT OF SPACE AT X=',F12.5)
STOP
END

```

```

SUBROUTINE CDFINV(C,P)
REAL*8 X,CDF,A,B,CC,DDC,ADINT1,XC,E
COMMON/CDF/X(2001),CDF(2001),A(2001),B(2001),NP1
CALL TABLOC(CDF,CBLE(C),NP1,I)
XC = X(I) + (C-CDF(I))/(CDF(I+1) - CDF(I))*(X(I+1) - X(I))
IF(DABS(A(I)).LE..5D-4)GO TO 410
300 DO 400 J=1,10
DC = ADINT1(X(I),XC,E,IER)
DDC = (CDF(I) + DC - C)/PDF(XC)
XC = XC - DDC
IF(DABS(DDC).LT..5D-4)GO TO 410
IF(XC.LE.X(I))GO TO 350
IF(XC.GE.X(I+1))GO TO 360
GO TO 400
350 XC = X(I) + (C - CDF(I))/DC*(XC - X(I))
GO TO 400
360 XC = X(I+1) + (C - CDF(I+1))/(CDF(I) + DC - CDF(I+1))*(X(I+1)-X
400 CONTINUE
WRITE(6,1)C,XC
1 FORMAT(' 10 ITERATIONS IN CDFINV, C AND XC =',2E12.4)
410 P = XC
RETURN
END

```

```

SUBROUTINE TABLOC(XT,X,M,NL)
REAL*8 XT(1),X
NT = ALOG(FLOAT(M))/ .301 + 1.
NU = M
NL = 1
DO 200 I=1,NT
NG = (NU + NL)/2
IF(X.GE.XT(NG))GC TC 100
NU = NG
GO TO 200
100 NL = NG
200 CONTINUE
RETURN
END

```

```

C
C
C
FUNCTION ADINT1(X1,X2,ERROR,IER)
THIS SUBROUTINE USES DCADRE TO OBTAIN THE INTEGRAL OF A
PEARSON TYPE I OR II DISTRIBUTION FUNCTION. SUBTRACTING
OUT THE SINGULARITY IS USED.

IMPLICIT REAL*8 (A-Z)
INTEGER*4 I,IER
EXTERNAL F1,F2,PCF
COMMON/PPARM/C0,C1,C2,A1,A2,BC0,BC1,M1,M2,KINV,XL,XR,MEAN,T
IF(X1.GT.(XL+XR)/2.C0)GO TO 200
IF(M1.GT.0.D0)GO TC 300
PR = DCADRE(F1,X1,X2,1.D-6,0.D0,ERROR,IER)
ADINT1 = PR + (A2-A1)**M2*((X2-A1)**(M1+1.D0)-(X1-A1)**(M1+1.D0)
1 / (M1 + 1.D0)/KINV
RETURN
200 IF(M2.GT.0.D0)GO TC 300
PR = DCADRE(F2,X1,X2,1.D-6,0.D0,ERROR,IER)
ADINT1 = PR-(A2 - A1)**M1*((A2-X2)**(M2+1.D0)-(A2-X1)**(M2+1.D0)

```



```

1 / (M2+1.D0)/KINV
RETURN
300 ADINT1 = DCADRE(PDF,X1,X2,1.D-6,0.D0,ERROR,IEP)
RETURN
END

```

```

FUNCTION F1(X)
IMPLICIT REAL*8 (A-Z)
COMMON /FPARM/CO,C1,C2,A1,A2,BC0,BC1,M1,M2,KINV,XL,XR,MEAN,T
IF(X.LE.A1)GO TO 200
F1 = (X-A1)**M1*((A2 - X)**M2 - (A2 - A1)**M2)/KINV
RETURN
200 F1 = 0.DC
RETURN
END

```

```

FUNCTION F2(X)
IMPLICIT REAL*8(A-Z)
COMMON/FPARM/CO,C1,C2,A1,A2,BC0,BC1,M1,M2,KINV,XL,XR,MEAN,T
IF(X.GE.A2)GO TO 200
F2 = (A2- X)**M2*((X-A1)**M1 - (A2-A1)**M1)/KINV
RETURN
200 F2 = 0.DC
RETURN
END

```

CCCCCCCCCCCC

```

SUBROUTINE FCUTZ (PCDF,XT,NXT,FN)
THIS SUBROUTINE GENERATES THE STATISTIC FOR THE FCUTZ FN TEST.
INPUT VARIABLES ARE:
  PCDF - THE CUMULATIVE DISTRIBUTION FUNCTION AGAINST WHICH THE
          DEVIATES ARE BEING TESTED. CALLING SEQUENCE MUST BE OF
          THE FORM 'CALL PCDF(X,P)', WHERE X IS AN INPUT VALUE,
          AND THE VALUE OF THE CUMULATIVE DISTRIBUTION FUNCTION
          IS RETURNED IN P.
          P MUST BE BETWEEN 0 AND 1.
  XT - THE ARRAY OF DEVIATES, IN INCREASING ORDER.
  NXT - THE NUMBER OF DEVIATES IN THE ARRAY XT (= N - 1)

```

THE RETURNED VALUE IS FN, THE VALUE OF THE STATISTIC.

CCCCCCCCCCCC

```

NXT IS PRESENTLY LIMITED TO A MAXIMUM OF 50 BY THE DIMENSION OF
THE VARIABLE XTD.
DIMENSION XT(1)
REAL*8 XTD(51),RN,FND
N = NXT + 1
DO 200 I=1,NXT
  K = N - I
  CALL PCDF(XT(K),P)
200 XTD(K+1) = P
  RN = 1.DC/N
  XTD(1) = RN - XTD(2)
DO 300 I=2,NXT
300 XTD(I) = RN - XTD(I+1) + XTD(I)
  XTD(N) = RN - 1.DC + XTD(N)
  FND = 0.
DO 400 I=1,N
400 FND = FND + DMAX1(XTD(I),0.D0)
  FN = FND
RETURN
END

```

APPENDIX 2

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	410	429	481
	5%	240	240	229
	1%	48	55	56
RSSF VS N (0,1)	10%	1784	2908	4125
	5%	1362	2384	3645
	1%	532	1410	2673

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	491	495	483
	5%	253	253	234
	1%	49	58	46
RSSF VS N (0,1)	10%	1636	2202	3318
	5%	1008	1401	2410
	1%	270	466	905

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	525	481	488
	5%	225	250	253
	1%	40	35	63
RSSF VS N (0,1)	10%	3603	4010	4597
	5%	2999	3515	4279
	1%	1894	2463	3488

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM
ALPHA = 1.00, BETA = 0.0

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	350	421	498
	5%	220	219	242
	1%	49	57	54
RSSF	10%	1460	2407	3616
VS	5%	1084	1889	3039
N (C,1)	1%	401	1011	1967

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CENTROL	10%	488	460	500
	5%	260	244	231
	1%	64	50	38
RSSF	10%	1412	1782	2654
VS	5%	792	1083	1760
N (0,1)	1%	222	324	579

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CENTROL	10%	503	494	501
	5%	256	257	241
	1%	60	43	42
RSSF	10%	2863	3272	3963
VS	5%	2145	2678	3422
N (0,1)	1%	1102	1575	2244

'CENTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.30, BETA = 0.0

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CENTFCL	10%	395	395	505
	5%	221	199	231
	1%	39	40	45
RSSF VS N (0,1)	10%	1441	2116	3268
	5%	1054	1598	2692
	1%	390	817	1663

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	493	472	502
	5%	245	215	265
	1%	50	47	52
RSSF VS N (0,1)	10%	1321	1635	2363
	5%	780	991	1508
	1%	210	294	531

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	468	449	498
	5%	188	222	237
	1%	37	51	40
RSSF VS N (0,1)	10%	2479	2678	3308
	5%	1804	2059	2705
	1%	852	1080	1573

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.60, EETA = 0.0

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	389	405	491
	5%	230	221	235
	1%	43	47	48
RSSF VS N (0,1)	10%	1326	1933	3046
	5%	932	1469	2428
	1%	320	688	1440

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	527	474	489
	5%	275	236	262
	1%	59	38	54
RSSF VS N (0,1)	10%	1236	1521	2184
	5%	760	929	1389
	1%	242	271	474

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CENTROL	10%	531	505	516
	5%	265	263	262
	1%	53	49	54
RSSF VS N (0,1)	10%	2046	2274	2716
	5%	1396	1646	2081
	1%	581	788	1092

'CENTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM
ALPHA = 1.90, BETA = 0.0

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	400	457	496
	5%	234	242	246
	1%	40	51	57
RSSF VS N (0,1)	10%	1728	2935	4056
	5%	1316	2435	3629
	1%	555	1454	2672

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	504	474	477
	5%	252	235	234
	1%	68	52	47
RSSF VS N (0,1)	10%	1565	2222	3221
	5%	966	1410	2372
	1%	310	481	901

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	493	523	467
	5%	238	255	250
	1%	49	53	46
RSSF VS N (0,1)	10%	3502	4014	4596
	5%	2859	3522	4255
	1%	1753	2547	3487

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM
ALPHA = 1.00, BETA = 0.25

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	372	416	465
	5%	225	232	204
	1%	42	56	45
RSSF VS N (0,1)	10%	1521	2451	3593
	5%	1128	1933	3106
	1%	454	1057	2046

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CENTRCL	10%	486	512	483
	5%	242	272	258
	1%	48	62	45
RSSF VS N (0,1)	10%	1430	1849	2701
	5%	890	1171	1882
	1%	268	371	675

FCUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	461	465	459
	5%	232	252	228
	1%	37	53	47
RSSF VS N (0,1)	10%	2868	3284	4026
	5%	2236	2684	3486
	1%	1174	1545	2325

'CENTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.20, EETA = 0.25

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	395	390	467
	5%	223	212	242
	1%	31	54	53
RSSF VS N (0,1)	10%	1416	2100	3277
	5%	1039	1610	2705
	1%	357	786	1607

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CENTROL	10%	514	518	504
	5%	260	282	278
	1%	50	61	65
RSSF VS N (0,1)	10%	1345	1661	2402
	5%	807	1017	1618
	1%	239	331	539

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CENTROL	10%	493	502	495
	5%	248	260	240
	1%	45	35	53
RSSF VS N (0,1)	10%	2383	2650	3260
	5%	1698	2023	2611
	1%	822	1027	1473

'CENTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.60, BETA = 0.25

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	402	441	497
	5%	222	230	237
	1%	42	45	48
RSSF VS N (0,1)	10%	1289	1916	3044
	5%	933	1458	2409
	1%	319	652	1328

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	488	469	487
	5%	234	235	242
	1%	40	52	53
RSSF VS N (0,1)	10%	1239	1471	2111
	5%	713	904	1357
	1%	223	269	445

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	484	470	494
	5%	215	246	242
	1%	56	50	49
RSSF VS N (0,1)	10%	2003	2289	2773
	5%	1358	1688	2098
	1%	552	770	1007

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM
ALPHA = 1.90, BETA = 0.25

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	383	427	455
	5%	213	248	253
	1%	45	56	37
RSSF	10%	1732	2905	4113
VS	5%	1308	2395	3668
N (0,1)	1%	539	1432	2660

KCLMCG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	507	492	484
	5%	245	233	232
	1%	38	47	37
RSSF	10%	1663	2188	3252
VS	5%	1011	1430	2414
N (0,1)	1%	289	455	930

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	507	492	534
	5%	259	237	271
	1%	53	58	50
RSSF	10%	3452	4011	4571
VS	5%	2399	3512	4297
N (0,1)	1%	1822	2489	3455

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.00, BETA = 0.50

NUMBER OF HYPOTHESES REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	382	406	443
	5%	213	195	207
	1%	45	38	38
RSSF VS N (0,1)	10%	1665	2552	3655
	5%	1266	2038	3222
	1%	509	1178	2202

KOLMOGOROV-SMIRNOV TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	501	503	486
	5%	248	256	253
	1%	45	55	57
RSSF VS N (0,1)	10%	1600	2092	3025
	5%	1026	1425	2208
	1%	342	532	951

FROOTZ-FRANK TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	506	467	471
	5%	246	236	228
	1%	44	46	43
RSSF VS N (0,1)	10%	2816	3278	3920
	5%	2168	2680	3382
	1%	1180	1546	2270

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM
ALPHA = 1.30, BETA = 0.50

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	413	440	470
	5%	251	222	205
	1%	36	48	41
RSSF VS N (0,1)	10%	1451	2269	3435
	5%	1063	1794	2841
	1%	366	948	1826

		KOLMOGOROV-SMIRNOV TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	478	518	513
	5%	252	278	277
	1%	49	65	60
RSSF VS N (0,1)	10%	1435	1853	2636
	5%	888	1229	1881
	1%	287	444	758

		FOOTZ FRIEDMAN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	493	463	488
	5%	241	220	227
	1%	47	47	40
RSSF VS N (0,1)	10%	2314	2643	3268
	5%	1686	2028	2594
	1%	761	1052	1449

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.60, BETA = 0.50

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	419	411	466
	5%	218	230	234
	1%	38	54	52
RSSF VS N (C,1)	10%	1299	1934	3042
	5%	916	1438	2407
	1%	297	680	1348

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	502	494	483
	5%	260	255	248
	1%	47	58	63
RSSF VS N (C,1)	10%	1239	1559	2148
	5%	738	961	1357
	1%	214	296	475

FCUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	473	479	508
	5%	259	229	240
	1%	55	53	52
RSSF VS N (0,1)	10%	2018	2310	2708
	5%	1384	1629	2043
	1%	591	749	1023

'CCNTFOL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE FANDOM STABILIZED STANDARD FORM

ALPHA = 1.90, BETA = 0.50

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTROL	10%	410	445	486
	5%	229	236	221
	1%	30	49	46
RSSF VS N (0,1)	10%	1831	2913	4108
	5%	1358	2391	3655
	1%	566	1438	2690

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTROL	10%	524	505	527
	5%	250	259	251
	1%	46	53	55
RSSF VS N (0,1)	10%	1627	2196	3304
	5%	1051	1421	2387
	1%	304	500	931

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CCNTROL	10%	515	497	502
	5%	229	254	255
	1%	51	59	46
RSSF VS N (0,1)	10%	3586	4057	4542
	5%	3005	3591	4256
	1%	1893	2495	3453

'CCNTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM
 ALPHA = 1.00, EETA = 0.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	368	447	505
	5%	200	241	223
	1%	43	58	39
RSSF VS N (0,1)	10%	1941	2911	4097
	5%	1495	2429	3670
	1%	704	1535	2762

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	475	504	524
	5%	242	237	264
	1%	47	45	47
RSSF VS N (0,1)	10%	2017	2640	3736
	5%	1331	1930	3089
	1%	546	904	1762

FCUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	461	513	490
	5%	226	248	248
	1%	45	54	55
RSSF VS N (0,1)	10%	2895	3312	4051
	5%	2237	2712	3542
	1%	1209	1595	2426

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.30, BETA = 0.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	400	407	501
	5%	243	225	256
	1%	48	51	46
RSSF VS N (0,1)	10%	1628	2451	3633
	5%	1240	1965	3099
	1%	509	1126	2110

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	491	509	476
	5%	227	244	243
	1%	48	54	50
RSSF VS N (0,1)	10%	1622	2104	3023
	5%	1050	1465	2330
	1%	379	588	1145

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	522	490	450
	5%	266	267	243
	1%	52	60	56
RSSF VS N (0,1)	10%	2426	2722	3367
	5%	1753	2067	2727
	1%	825	1058	1614

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.60, BETA = 0.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	405	407	469
	5%	238	229	215
	1%	54	49	46
RSSF VS N (0,1)	10%	1366	1989	3088
	5%	987	1490	2487
	1%	341	704	1419

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	501	505	483
	5%	246	268	245
	1%	57	53	46
RSSF VS N (0,1)	10%	1273	1558	2246
	5%	797	936	1465
	1%	234	308	508

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	497	457	478
	5%	242	227	242
	1%	58	48	48
RSSF VS N (0,1)	10%	2102	2256	2745
	5%	1479	1615	2081
	1%	647	760	1076

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.90, BETA = 0.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	395	441	495
	5%	213	238	243
	1%	35	53	62
RSSF VS N (0,1)	10%	1793	2920	4109
	5%	1395	2386	3658
	1%	554	1413	2707

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	493	504	457
	5%	241	231	238
	1%	50	43	43
RSSF VS N (0,1)	10%	1640	2203	3325
	5%	1004	1392	2434
	1%	295	421	939

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	526	488	481
	5%	260	247	223
	1%	54	58	50
RSSF VS N (0,1)	10%	3530	4001	4577
	5%	2933	3501	4284
	1%	1849	2405	3405

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.00, BETA = 1.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	363	428	518
	5%	218	226	243
	1%	45	44	55

RSSF	10%	2651	3826	4682
VS	5%	2291	3457	4472
N (0,1)	1%	1344	2572	3935

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	526	533	524
	5%	243	268	282
	1%	51	48	70

RSSF	10%	2937	3776	4638
VS	5%	2326	3235	4378
N (0,1)	1%	1245	2035	3452

		FUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	488	477	500
	5%	259	216	238
	1%	54	53	44

RSSF	10%	2962	3429	4162
VS	5%	2260	2813	3660
N (0,1)	1%	1237	1705	2550

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.20, EETA = 1.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNT FOL	10%	412	443	472
	5%	230	237	220
	1%	30	50	46
RSSF	10%	1868	2768	3558
VS	5%	1487	2294	3541
N (C,1)	1%	701	1427	2578

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTROL	10%	513	477	514
	5%	268	251	251
	1%	56	56	49
RSSF	10%	1968	2603	3652
VS	5%	1370	1903	3016
N (0,1)	1%	586	905	1765

		FOUZZY FN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	507	504	505
	5%	250	240	252
	1%	58	45	58
RSSF	10%	2414	2792	3409
VS	5%	1786	2120	2771
N (C,1)	1%	853	1123	1625

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM
 ALPHA = 1.60, BETA = 1.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	374	431	525
	5%	231	232	263
	1%	48	57	58
RSSF	10%	1339	1553	3107
VS	5%	977	1438	2466
N (0,1)	1%	347	708	1459

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	494	481	516
	5%	239	240	241
	1%	54	47	43
RSSF	10%	1322	1625	2308
VS	5%	808	978	1533
N (C,1)	1%	235	302	540

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	525	523	520
	5%	236	271	296
	1%	50	67	55
RSSF	10%	2086	2280	2758
VS	5%	1409	1645	2075
N (C,1)	1%	620	715	1078

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'RSSF' IS THE RANDOM STABILIZED STANDARD FORM

ALPHA = 1.90, BETA = 1.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	388	469	486
	5%	229	238	224
	1%	39	48	43
MIXNCRM VS N (C,1)	10%	1272	1925	3004
	5%	909	1407	2381
	1%	329	664	1337

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	529	513	510
	5%	254	260	249
	1%	42	46	64
MIXNCRM VS N (C,1)	10%	1235	1537	2133
	5%	713	931	1369
	1%	208	285	449

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	508	476	523
	5%	245	222	255
	1%	50	38	60
MIXNCRM VS N (0,1)	10%	1982	2191	2610
	5%	1336	1576	1995
	1%	533	672	977

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$C.C * N(.0,1) + 1.00 * N(C,2)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	409	411	498
	5%	233	205	234
	1%	29	34	46
MIXNCRM VS N (0,1)	10%	2513	3558	4620
	5%	2088	3080	4342
	1%	1028	2052	3609

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	475	463	500
	5%	238	229	242
	1%	46	35	38
MIXNCRM VS N (0,1)	10%	2188	2827	3965
	5%	1420	1974	3161
	1%	512	727	1515

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	522	454	481
	5%	236	217	226
	1%	47	41	52
MIXNCRM VS N (0,1)	10%	3338	3712	4346
	5%	2724	3174	3935
	1%	1657	2064	2970

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$$C.C * N(.0,1) + 1.00 * N(0,3)$$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	407	425	511
	5%	234	232	252
	1%	48	48	50
MIXNORM VS N (0,1)	10%	3427	4423	4935
	5%	3006	4154	4863
	1%	1781	3377	4639

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	510	490	511
	5%	261	241	250
	1%	59	48	47
MIXNORM VS N (0,1)	10%	2976	3799	4717
	5%	2113	3025	4286
	1%	890	1471	2792

FOITZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	518	501	528
	5%	242	268	276
	1%	68	66	62
MIXNORM VS N (0,1)	10%	4131	4502	4865
	5%	3686	4195	4742
	1%	2653	3380	4272

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

$0.0 * N(0,1) + 1.00 * N(0,4)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	382	422	510
	5%	215	187	254
	1%	41	38	46
MIXNCRM VS N (C,1)	10%	472	542	713
	5%	253	308	370
	1%	29	66	74

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	488	537	525
	5%	238	250	265
	1%	46	50	53
MIXNCRM VS N (C,1)	10%	587	667	695
	5%	287	328	367
	1%	54	70	84

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	468	480	517
	5%	220	258	247
	1%	46	64	56
MIXNCRM VS N (C,1)	10%	799	796	881
	5%	429	426	477
	1%	105	107	136

'CCNTFOL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$C.7C * N(.0,1) + C.30 * N(0,2)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	421	457	510
	5%	245	244	221
	1%	49	52	38
MIXNCRM VS N (C,1)	10%	551	727	1047
	5%	345	438	602
	1%	82	131	185

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTROL	10%	490	504	527
	5%	251	256	235
	1%	46	58	42
MIXNORM VS N (0,1)	10%	646	745	845
	5%	337	405	447
	1%	81	93	83

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	519	494	507
	5%	240	242	256
	1%	55	46	51
MIXNCFM VS N (0,1)	10%	1029	1139	1249
	5%	591	671	778
	1%	172	193	262

'CCNTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

$0.70 * N(0,1) + 0.30 * N(0,3)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	400	435	515
	5%	237	228	240
	1%	45	50	52
MIXNCRM VS N (0,1)	10%	636	910	1320
	5%	405	564	836
	1%	109	175	322

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	508	531	508
	5%	259	259	248
	1%	54	51	54
MIXNCRM VS N (0,1)	10%	747	853	1001
	5%	407	441	532
	1%	115	122	148

FOUITZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	492	478	511
	5%	229	232	241
	1%	61	47	43
MIXNCRM VS N (0,1)	10%	1281	1418	1666
	5%	774	888	1121
	1%	238	322	401

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

$0.70 * N(0,1) + 0.30 * N(0,4)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	380	431	507
	5%	221	245	220
	1%	40	59	56
MIXNCRM VS N (0,1)	10%	388	484	545
	5%	214	274	275
	1%	40	63	62

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	468	480	481
	5%	220	236	232
	1%	53	46	38
MIXNCRM VS N (0,1)	10%	519	539	556
	5%	258	277	286
	1%	58	55	53

		FOUTZ FM TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	487	498	502
	5%	234	242	265
	1%	42	49	54
MIXNCFM VS N (0,1)	10%	684	689	687
	5%	352	354	400
	1%	83	92	96

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$C.8C * N(.0,1) + 0.20 * N(0,2)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTFOL	10%	417	405	492
	5%	218	211	243
	1%	43	38	42
MIXNCRM VS N (0,1)	10%	477	544	736
	5%	281	306	364
	1%	48	64	98

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CENTRCL	10%	500	469	537
	5%	267	222	260
	1%	47	46	46
MIXNCRM VS N (0,1)	10%	587	590	678
	5%	317	297	343
	1%	69	59	78

FCUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	495	462	479
	5%	234	213	245
	1%	49	44	48
MIXNCRM VS N (0,1)	10%	839	819	945
	5%	475	454	522
	1%	118	130	143

'CENTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$0.80*N(0,1) + 0.20*N(0,3)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	402	450	539
	5%	225	242	247
	1%	43	49	60
MIXNCRM VS N (0,1)	10%	507	637	902
	5%	310	361	456
	1%	74	89	138

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	511	502	543
	5%	253	252	270
	1%	51	43	63
MIXNCRM VS N (0,1)	10%	622	661	773
	5%	329	360	416
	1%	85	69	93

		FCUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	485	496	514
	5%	238	243	255
	1%	48	48	55
MIXNCRM VS N (0,1)	10%	958	1011	1168
	5%	548	583	721
	1%	171	174	246

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION
 $0.8 * N(0,1) + 0.20 * N(0,4)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	420	470	470
	5%	231	240	238
	1%	40	51	44
MIXNORM VS N (0,1)	10%	410	478	535
	5%	228	241	247
	1%	35	48	46

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	499	490	519
	5%	244	245	251
	1%	54	44	55
MIXNORM VS N (0,1)	10%	531	512	535
	5%	264	257	266
	1%	57	47	64

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	493	490	480
	5%	245	256	229
	1%	43	62	47
MIXNORM VS N (0,1)	10%	577	583	587
	5%	282	310	290
	1%	66	73	77

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION
 $0.90 * N(0,1) + 0.10 * N(0,2)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	432	416	486
	5%	246	214	215
	1%	43	41	40
MIXNCRM VS N (0,1)	10%	456	431	537
	5%	262	233	258
	1%	51	43	53

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	509	495	509
	5%	271	239	252
	1%	55	46	55
MIXNCRM VS N (0,1)	10%	553	526	557
	5%	302	261	274
	1%	67	53	64

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	516	471	520
	5%	271	239	252
	1%	58	51	47
MIXNCRM VS N (0,1)	10%	660	653	675
	5%	353	357	356
	1%	95	88	89

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$0.90*N(0,1) + 0.10*N(0,3)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	397	439	509
	5%	240	244	246
	1%	31	53	55
MIXNCRM VS N (0,1)	10%	419	508	551
	5%	260	266	302
	1%	36	65	74

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	484	516	488
	5%	240	247	239
	1%	56	47	49
MIXNCRM VS N (0,1)	10%	525	584	548
	5%	268	282	277
	1%	64	61	60

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CONTROL	10%	514	511	491
	5%	247	253	245
	1%	48	50	58
MIXNCRM VS N (0,1)	10%	736	707	747
	5%	385	384	422
	1%	81	105	112

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$C = 0.90 * N(.0, 1) + 0.10 * N(0, 4)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	386	417	454
	5%	225	241	238
	1%	39	44	46
MIXNCRM VS N (C,1)	10%	1459	2008	3146
	5%	1053	1533	2556
	1%	407	793	1568

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	493	483	487
	5%	267	246	238
	1%	52	53	38
MIXNCRM VS N (C,1)	10%	2179	2806	3756
	5%	1606	2181	3226
	1%	714	1148	2043

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	535	495	499
	5%	250	246	248
	1%	54	43	41
MIXNCRM VS N (0,1)	10%	1809	1994	2492
	5%	1198	1428	1825
	1%	509	602	885

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNORM' IS THE MIXED NORMAL DISTRIBUTION

$0.70*N(.5,1) + 0.30*N(C,3)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	377	449	512
	5%	205	239	257
	1%	46	46	44
MIXNCRM VS N (0,1)	10%	1557	2112	3304
	5%	1170	1563	2660
	1%	473	750	1640

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	482	481	538
	5%	229	260	271
	1%	45	69	56
MIXNCRM VS N (0,1)	10%	2450	3101	4056
	5%	1855	2510	3596
	1%	901	1344	2400

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	479	512	531
	5%	223	262	255
	1%	50	53	53
MIXNCRM VS N (0,1)	10%	1781	1917	2405
	5%	1209	1369	1774
	1%	462	573	843

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$0.80 * N(.5, 1) + 0.20 * N(0, 3)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	389	439	490
	5%	221	251	221
	1%	38	55	49
MIXNCRM VS N (0,1)	10%	1568	2228	3377
	5%	1177	1736	2832
	1%	507	951	1796

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	535	489	490
	5%	258	253	249
	1%	55	54	35
MIXNCRM VS N (0,1)	10%	2467	3194	4105
	5%	1865	2593	3659
	1%	857	1438	2511

		FCUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	502	496	495
	5%	252	250	239
	1%	62	51	51
MIXNCRM VS N (0,1)	10%	1838	2156	2665
	5%	1220	1551	1978
	1%	486	701	979

'CONTROL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'MIXNCRM' IS THE MIXED NORMAL DISTRIBUTION

$0.8 * N(0.5, 1) + 0.20 * N(0, 4)$

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	431	413	528
	5%	240	200	249
	1%	40	42	53
PEARSON VS N (0,1)	10%	465	478	626
	5%	258	242	302
	1%	46	59	71

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	528	478	525
	5%	275	242	283
	1%	45	49	54
PEARSON VS N (0,1)	10%	623	571	673
	5%	339	310	371
	1%	66	70	95

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	512	458	530
	5%	248	207	263
	1%	56	48	58
PEARSON VS N (0,1)	10%	481	428	497
	5%	241	196	234
	1%	60	36	57

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.0 , BETA 2 = 2.30

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	435	443	541
	5%	237	241	247
	1%	49	52	49
PEARSON VS N (0,1)	10%	432	445	533
	5%	243	247	241
	1%	43	55	52

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	489	528	489
	5%	238	252	245
	1%	55	50	64
PEARSON VS N (0,1)	10%	496	543	503
	5%	242	274	262
	1%	61	54	67

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	536	467	487
	5%	275	227	240
	1%	61	34	43
PEARSON VS N (0,1)	10%	523	441	467
	5%	269	217	228
	1%	58	33	43

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.0 , BETA 2 = 2.80

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	408	390	453
	5%	230	214	213
	1%	35	50	52
FEARSCN VS N (0,1)	10%	756	992	1576
	5%	495	634	973
	1%	121	185	343

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CONTROL	10%	492	515	520
	5%	252	255	264
	1%	55	49	52
FEARSCN VS N (0,1)	10%	942	1161	1498
	5%	546	693	944
	1%	158	192	294

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	505	478	504
	5%	223	261	255
	1%	50	56	48
FEARSCN VS N (0,1)	10%	853	1400	2487
	5%	469	884	1785
	1%	129	311	799

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'FEARSCN' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.01, BETA 2 = 1.75

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	428	410	493
	5%	239	205	233
	1%	40	43	52
PEARSON VS N (C,1)	10%	614	744	1083
	5%	375	435	610
	1%	93	120	193

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	543	470	506
	5%	290	229	274
	1%	41	50	48
PEARSON VS N (C,1)	10%	837	857	1106
	5%	482	479	648
	1%	131	131	184

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTFOL	10%	511	474	515
	5%	240	214	259
	1%	55	49	54
PEARSON VS N (C,1)	10%	558	726	1288
	5%	305	405	774
	1%	82	101	233

'CCNTFOL' IS THE TEST OF UNIFORM(C,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.01, BETA 2 = 1.90

NUMBER OF HYPCTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	408	442	510
	5%	230	237	241
	1%	48	41	47
PEARSON VS N (C,1)	10%	458	556	702
	5%	267	321	366
	1%	51	74	96

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	492	485	502
	5%	240	272	258
	1%	55	50	65
PEARSON VS N (C,1)	10%	581	650	715
	5%	309	329	389
	1%	74	73	94

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	529	479	546
	5%	241	241	285
	1%	62	56	54
PEARSON VS N (C,1)	10%	534	554	686
	5%	269	283	353
	1%	68	66	81

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.25, BETA 2 = 3.20

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	373	419	502
	5%	220	230	211
	1%	55	49	34
PEARSON VS N (C,1)	10%	739	1030	1672
	5%	476	664	1099
	1%	118	231	421

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	516	483	475
	5%	253	227	231
	1%	48	51	38
PEARSON VS N (C,1)	10%	790	896	1125
	5%	474	528	735
	1%	121	147	234

FOUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	499	481	477
	5%	237	238	253
	1%	46	49	50
PEARSON VS N (C,1)	10%	868	1090	1518
	5%	471	660	965
	1%	121	185	344

'CCNTFCL' IS THE TEST OF UNIFORM(0,1).VS UNIFORM(C,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 0.50, BETA 2 = 3.00

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	401	428	522
	5%	236	225	239
	1%	43	39	54
PEARSON VS N (C,1)	10%	2440	3386	4654
	5%	1960	2820	4320
	1%	931	1632	3267

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	482	513	509
	5%	239	254	245
	1%	47	52	61
PEARSON VS N (C,1)	10%	1155	1459	2001
	5%	730	920	1428
	1%	216	322	609

		FOUITZ FN TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	531	492	550
	5%	250	250	277
	1%	57	51	61
PEARSON VS N (C,1)	10%	2135	2837	3920
	5%	1460	2156	3309
	1%	583	1049	2104

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 1.00, BETA 2 = 3.40

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

CHI SQUARED TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	411	427	475
	5%	226	238	227
	1%	34	49	34
PEARSON VS N (C,1)	10%	1446	3151	4513
	5%	1070	2546	4035
	1%	403	1368	2895

KOLMOG-SMIR TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	499	495	504
	5%	249	260	269
	1%	55	64	60
PEARSON VS N (C,1)	10%	1032	1314	1765
	5%	649	832	1227
	1%	211	265	493

FCUTZ FN TEST

		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	482	481	485
	5%	251	251	207
	1%	51	45	43
PEARSON VS N (C,1)	10%	1578	2161	3119
	5%	1009	1493	2354
	1%	348	633	1254

'CCNTFCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(C,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 1.00, BETA 2 = 3.60

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	397	427	489
	5%	232	233	234
	1%	41	60	56
PEARSON VS N (0,1)	10%	1086	2009	3236
	5%	764	1463	2543
	1%	255	651	1382

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	524	501	499
	5%	267	250	252
	1%	51	56	62
PEARSON VS N (0,1)	10%	911	1150	1528
	5%	546	732	1032
	1%	171	229	399

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CCNTFCL	10%	521	460	529
	5%	258	248	260
	1%	50	63	46
PEARSON VS N (0,1)	10%	1261	1656	2435
	5%	764	1070	1706
	1%	243	387	739

'CCNTFCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 1.00, BETA 2 = 3.80

NUMBER OF HYPOTHESIS REJECTIONS IN 5000 REPLICATIONS

		CHI SQUARED TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	371	431	464
	5%	206	237	227
	1%	37	48	45
PEARSON VS N (0,1)	10%	2231	3039	4476
	5%	1745	2383	3538
	1%	763	1242	2652

		KOLMOG-SMIR TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	498	482	517
	5%	250	261	261
	1%	42	48	54
PEARSON VS N (0,1)	10%	1153	1481	2082
	5%	753	1005	1447
	1%	231	353	621

		FOUTZ FN TEST		
		20 PTS	30 PTS	50 PTS
CCNTRCL	10%	450	498	486
	5%	258	255	248
	1%	44	45	61
PEARSON VS N (0,1)	10%	1872	2587	3548
	5%	1230	1920	2894
	1%	469	886	1644

'CCNTRCL' IS THE TEST OF UNIFORM(0,1) VS UNIFORM(0,1)

'PEARSON' IS THE PEARSON DISTRIBUTION

BETA 1 = 2.00, BETA 2 = 5.50

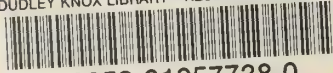
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